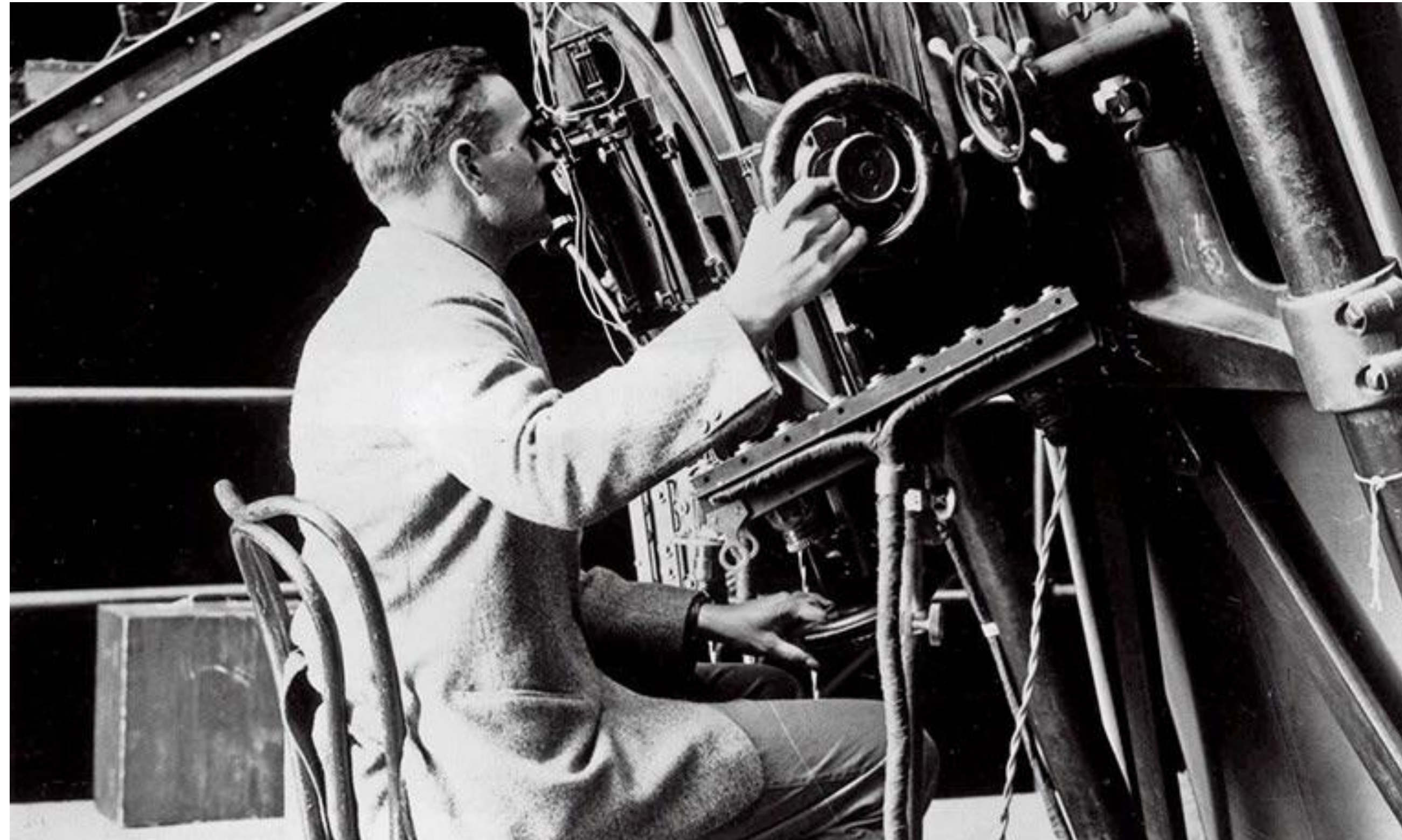
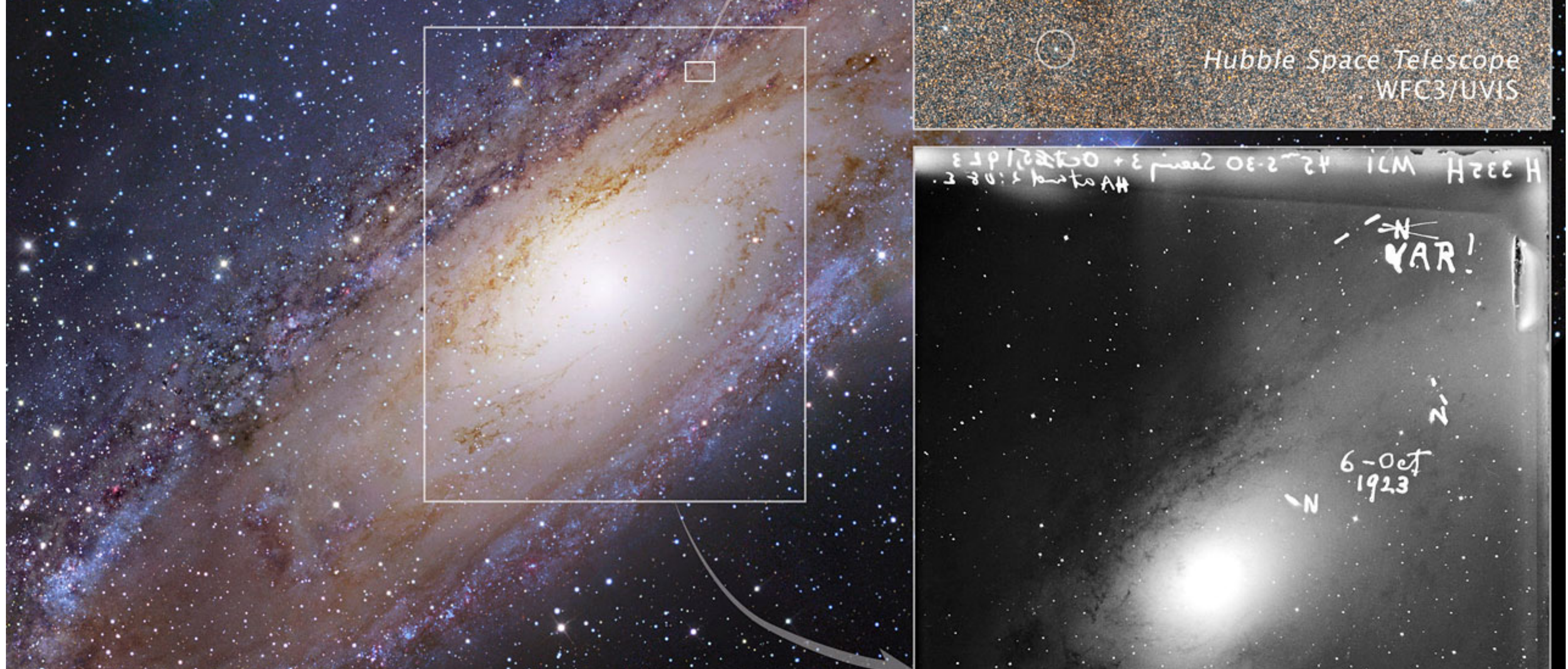


Hubble Trouble



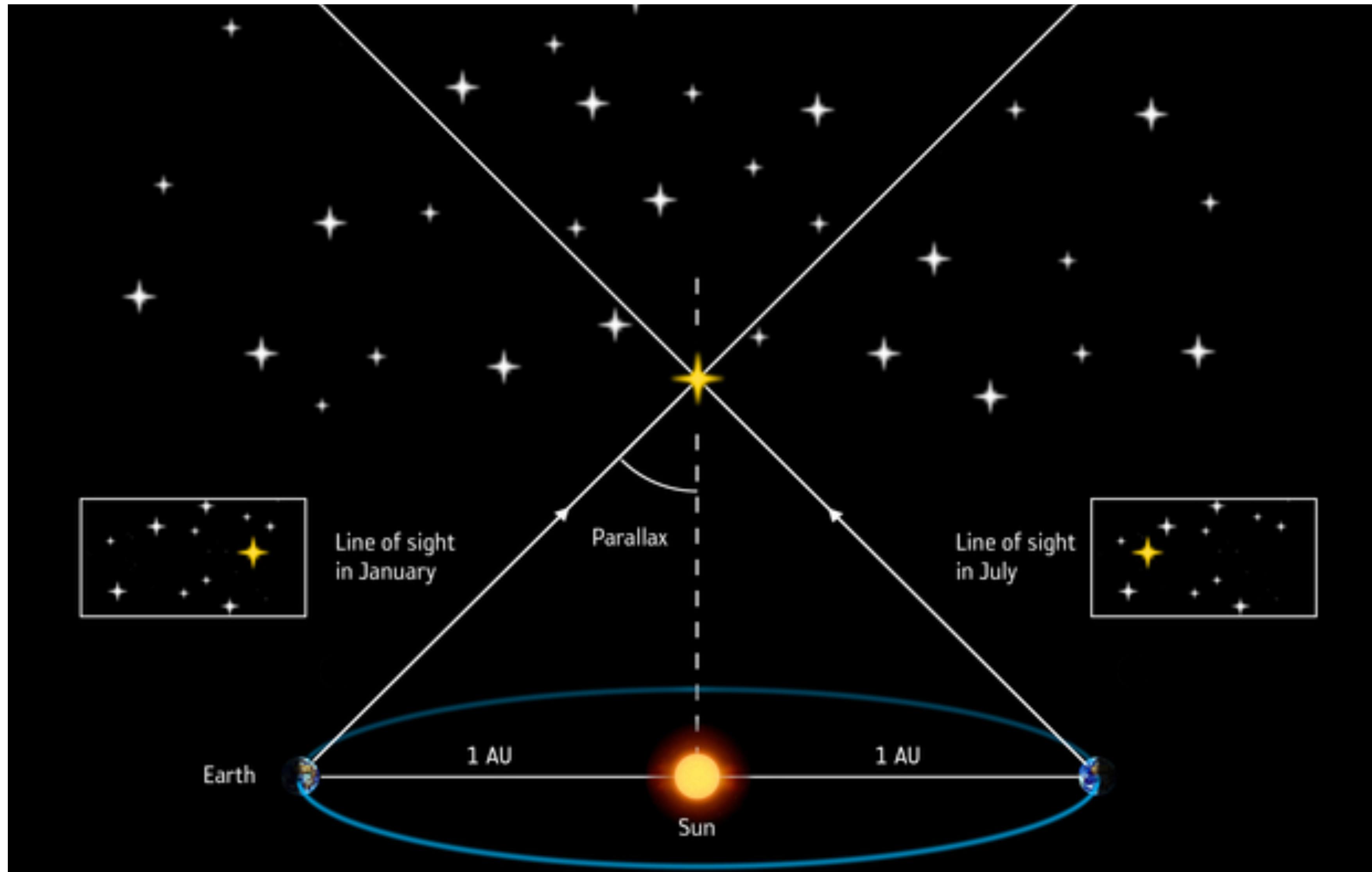
Edwin Hubble in 1922 at the controls of the new 100 inch Mt Wilson Hooker Telescope completed in 1917 and largest telescope until construction of the Hale Telescope on Mt Palomar in 1948



In 1923 Hubble finds a Cepheid Variable star in a nebula called M31 or the Andromeda Galaxy. The Wattage (luminosity) of this star can be found just by measuring its period. The apparent brightness of this Cepheid helped him work out the distance to M31 as 1 MLy, now revised to 2.5 MLy. Not taking dust into consideration, Shapley estimated our own Milky Way galaxy was 300,000 Ly across (now revised to 100 000 Ly). M31 was too far away to be inside our own Milky Way galaxy!

Finding Distances to Nearby Stars

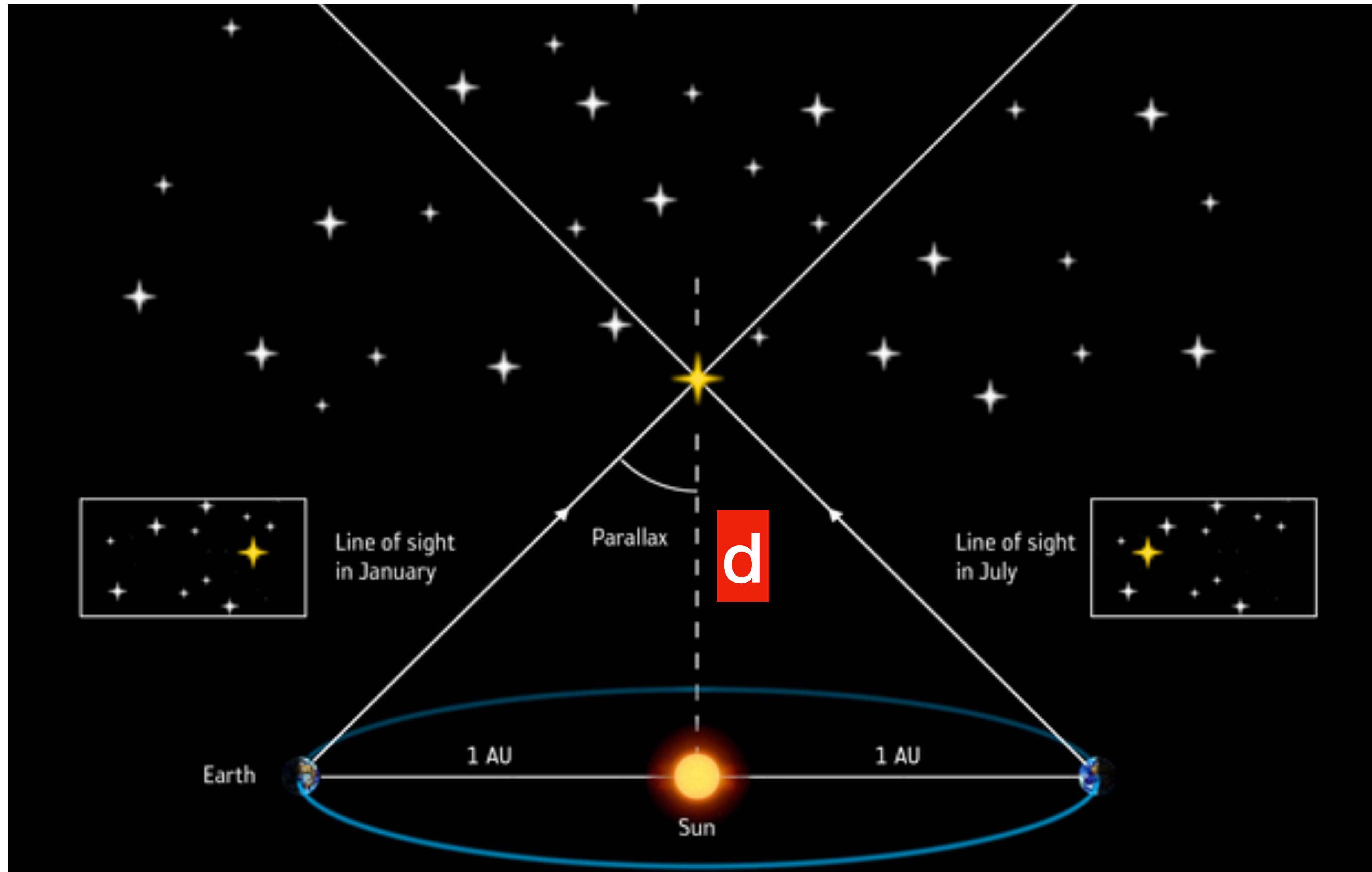
First rung in the Cosmic Ladder (Zero Point) Parallax



Finding Distances to Nearby Stars

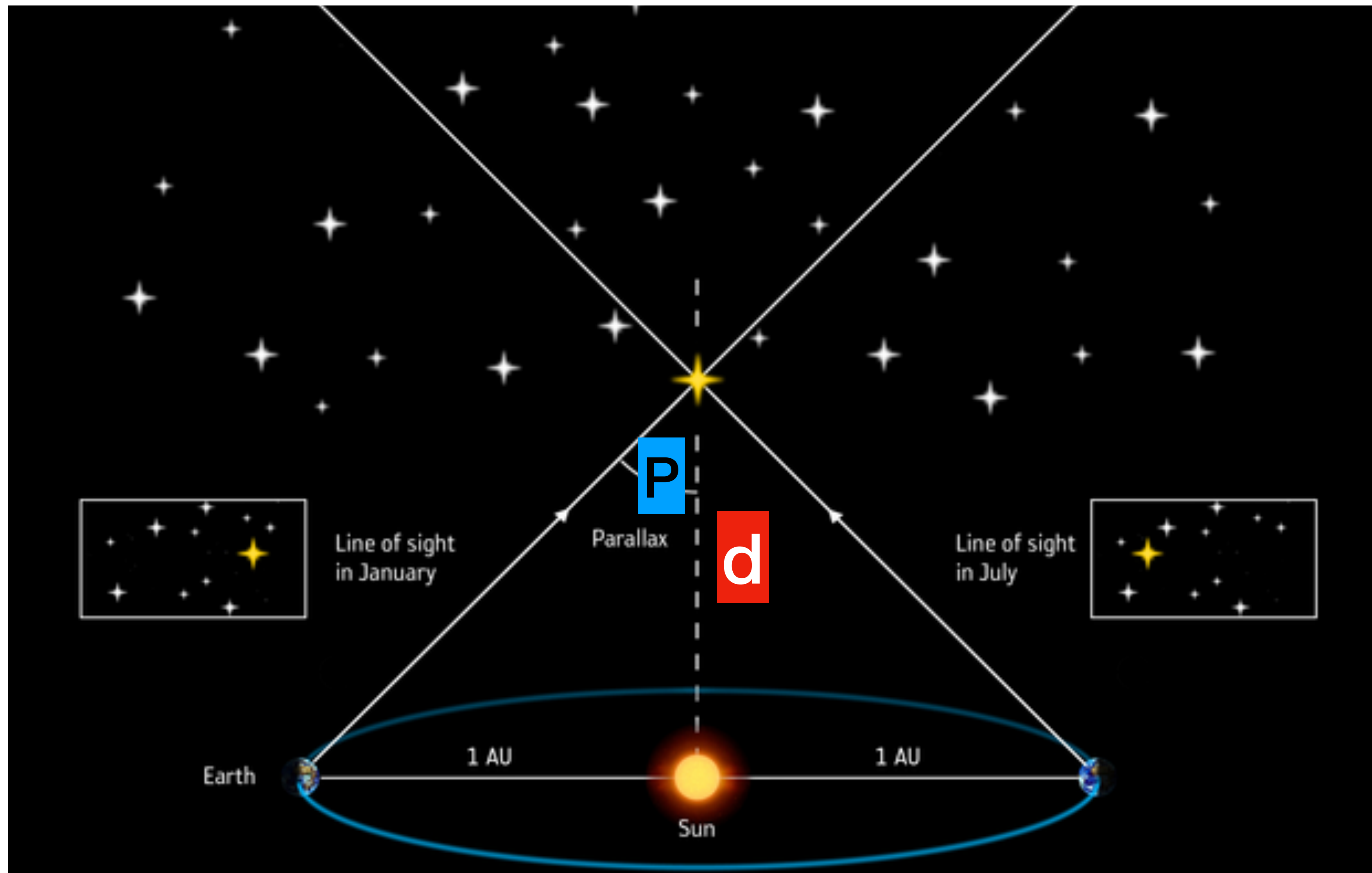
First rung in the Cosmic Ladder (Zero Point) Parallax

Distance to
nearby star
is “d”



Finding Distances to Nearby Stars

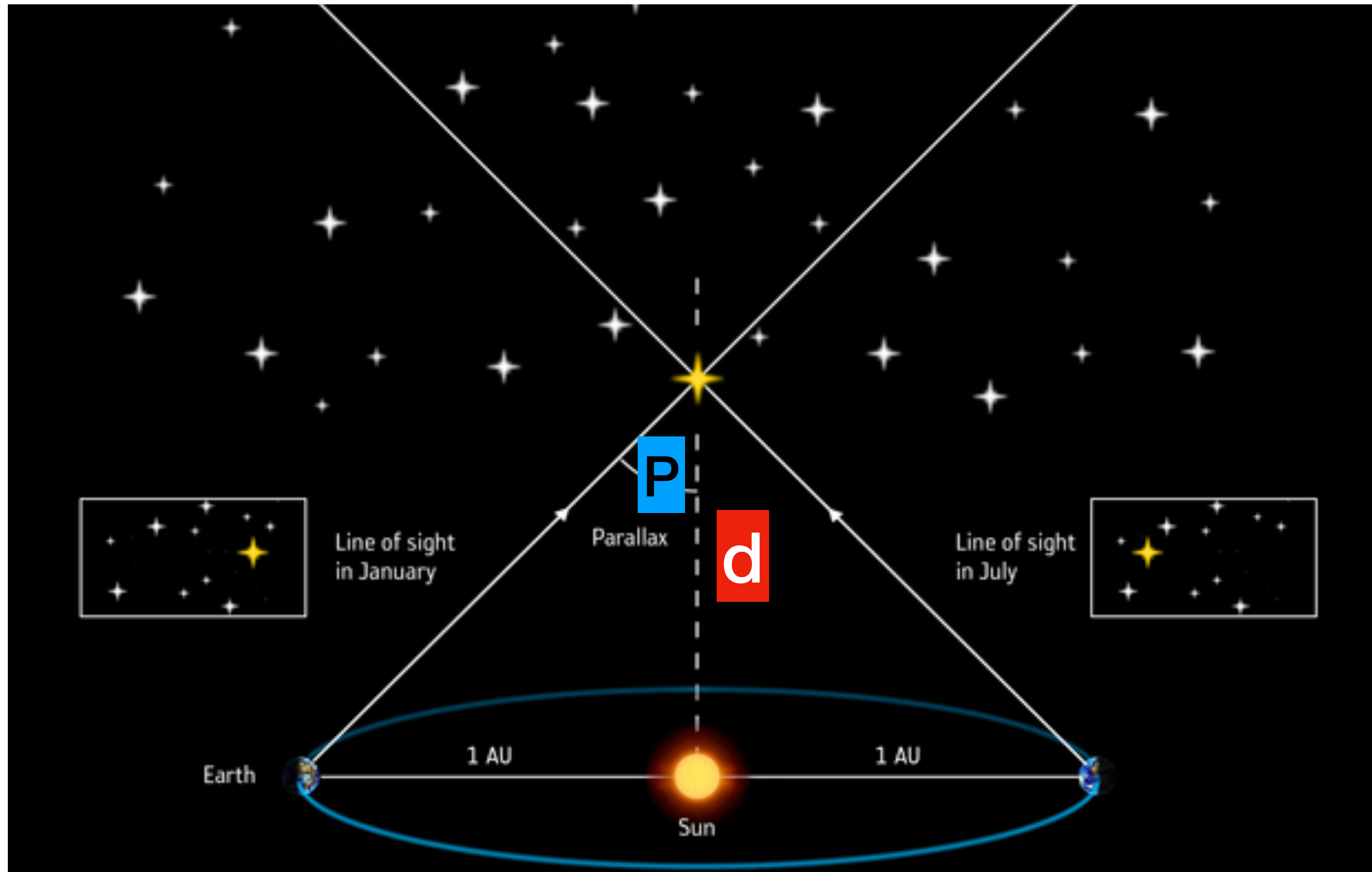
First rung in the Cosmic Ladder (Zero Point) **Parallax**



Parallax angle
is p

Finding Distances to Nearby Stars

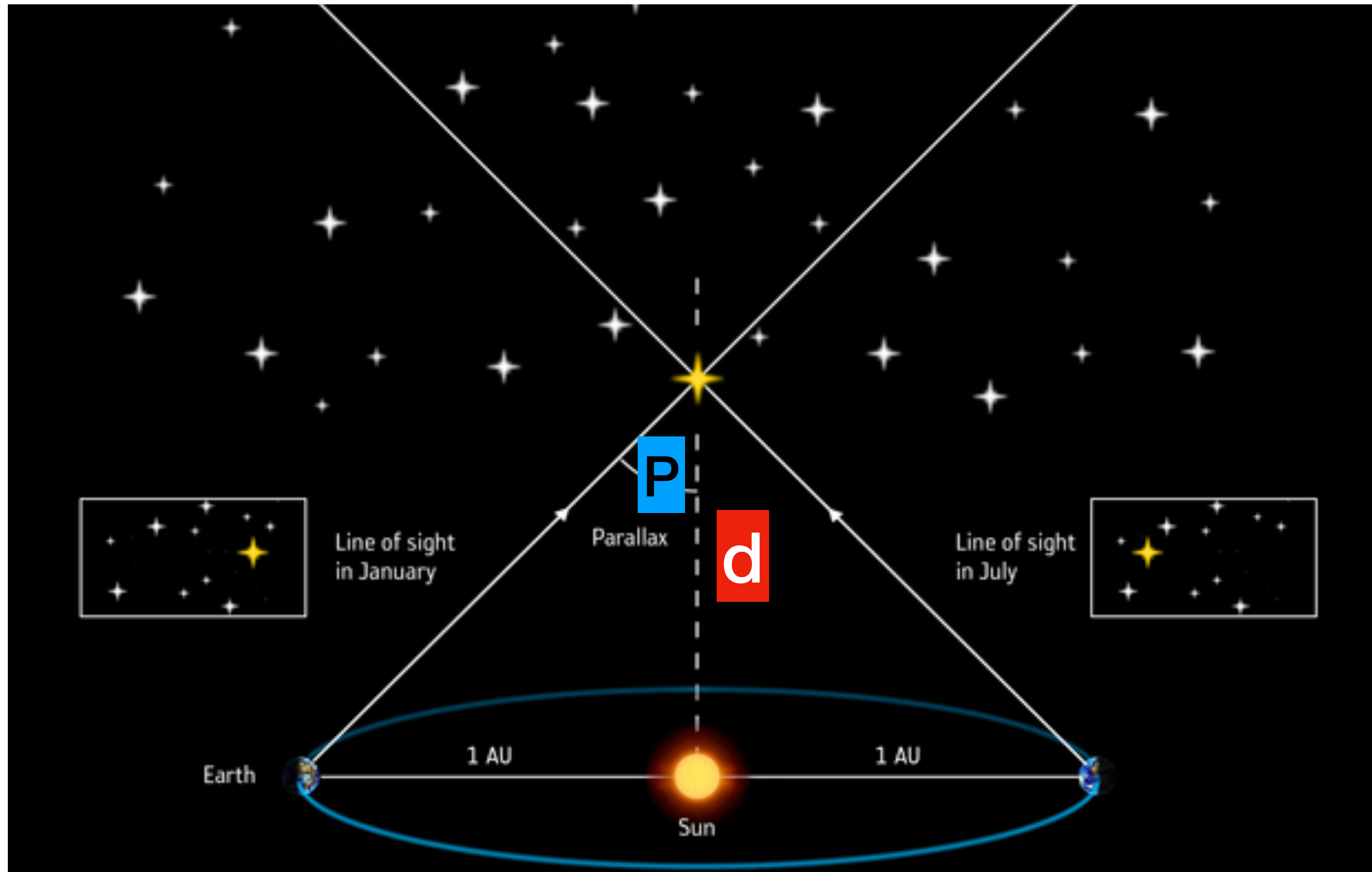
First rung in the Cosmic Ladder (Zero Point) Parallax



$$\tan p = 1 \text{ A.U.} / d$$

Finding Distances to Nearby Stars

First rung in the Cosmic Ladder (Zero Point) Parallax

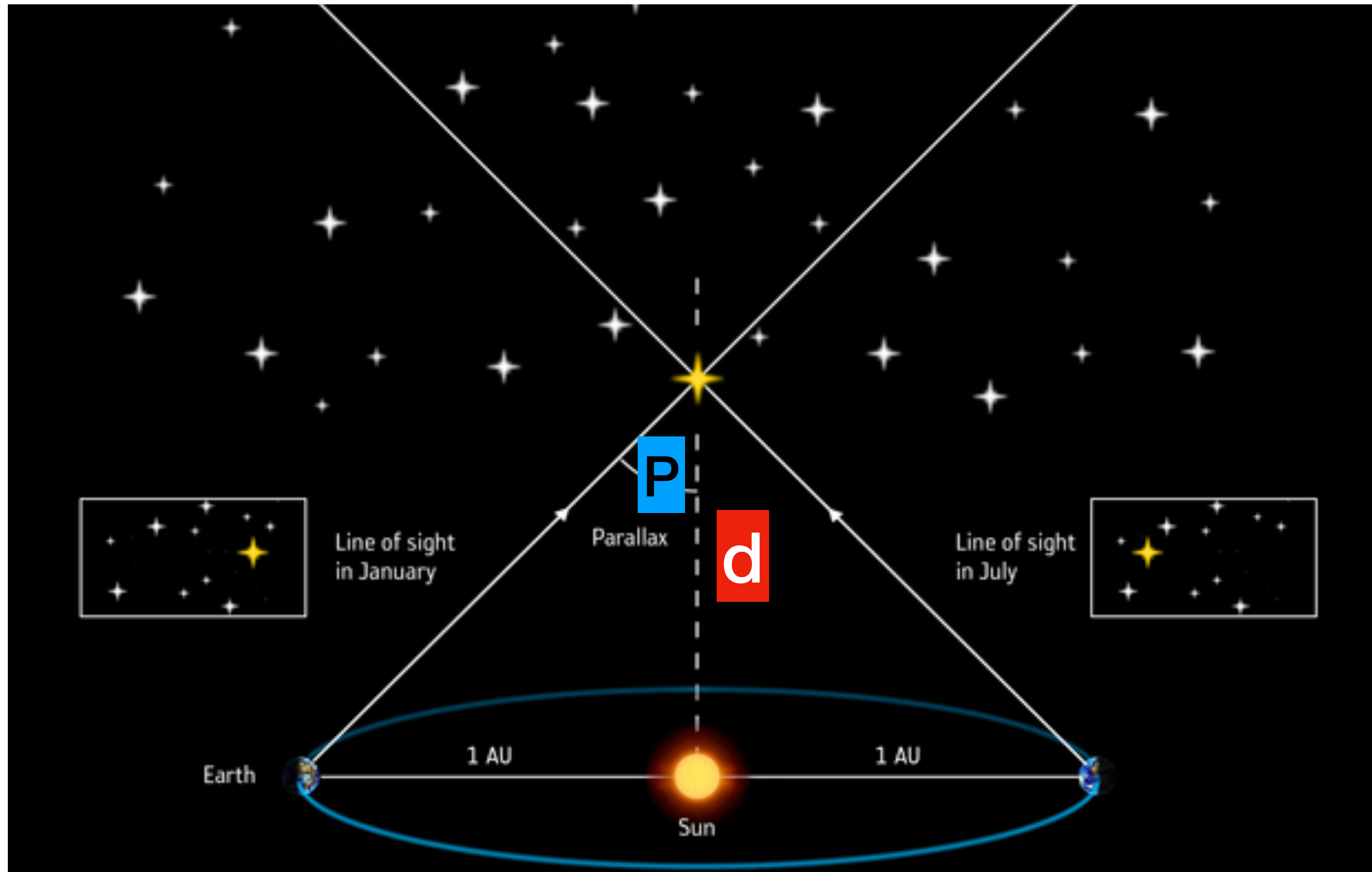


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For small angles, $\tan p = p$

Finding Distances to Nearby Stars

First rung in the Cosmic Ladder (Zero Point) Parallax



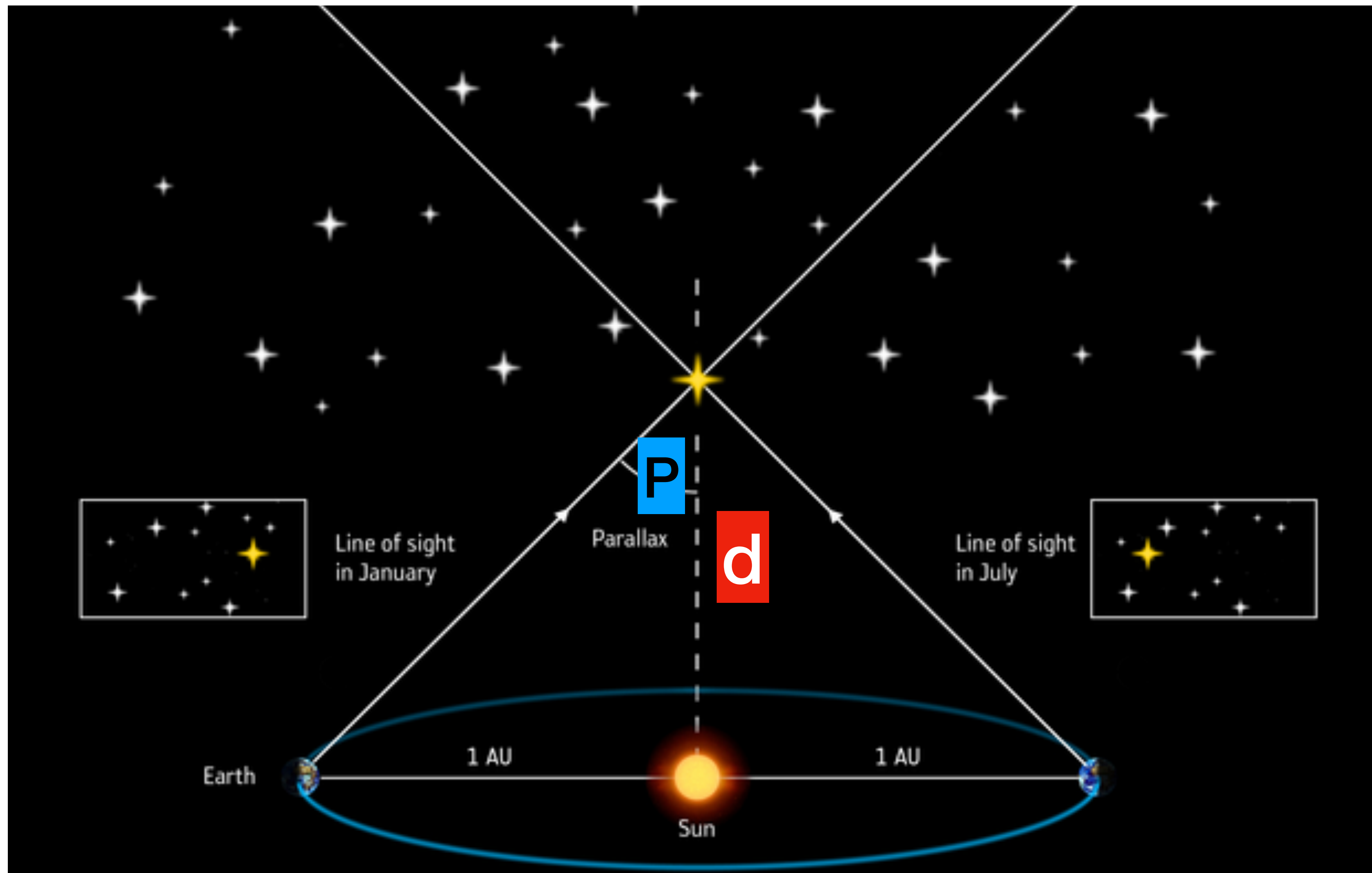
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First rung in the Cosmic Ladder (Zero Point) Parallax



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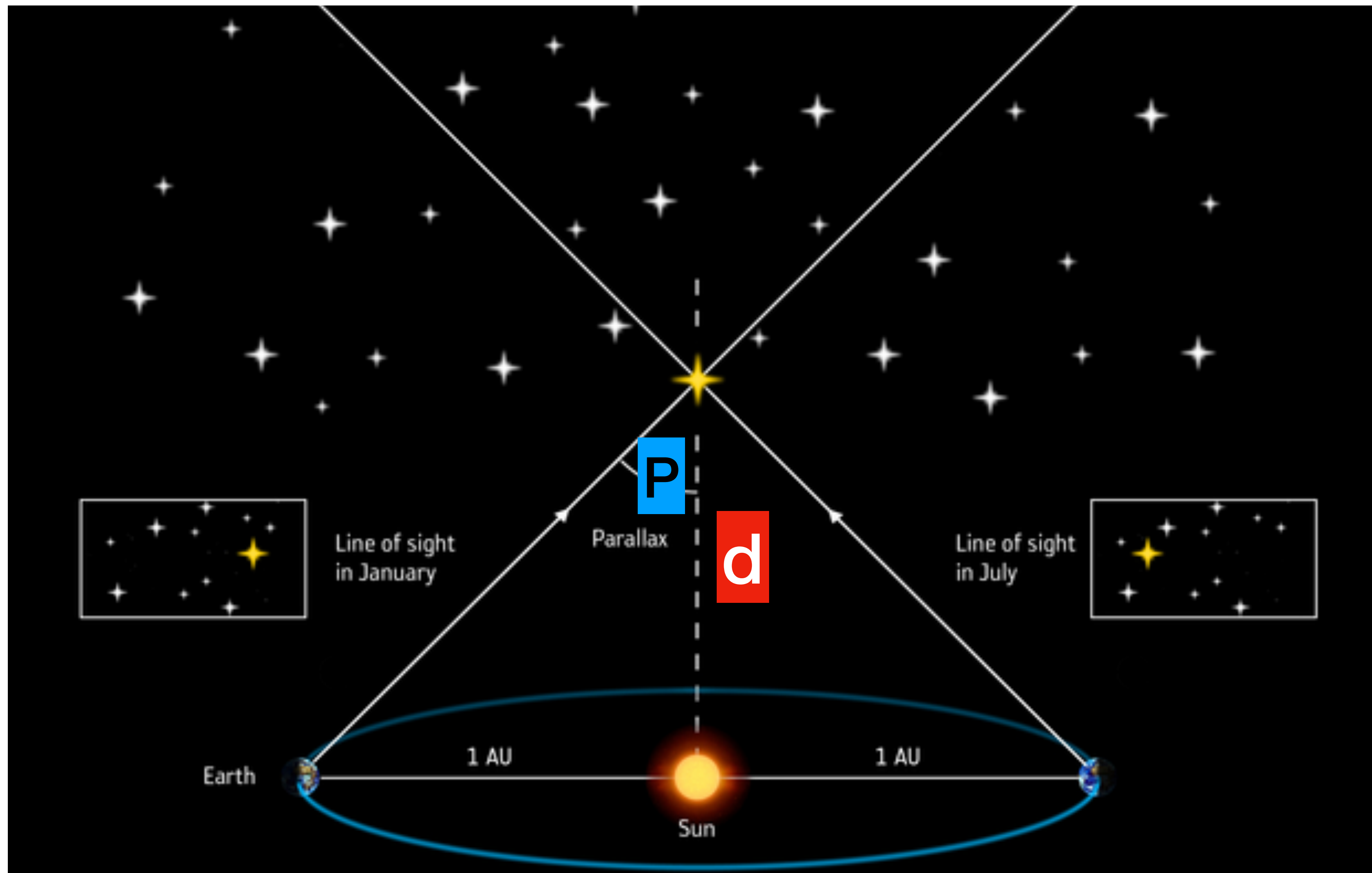
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$$P = 1 \text{ A.U.}/d$$

Unit = 1 second
= 1/60 X 1/60 degree

Finding Distances to Nearby Stars

First rung in the Cosmic Ladder (Zero Point) **Parallax**



$$\tan p = 1 \text{ A.U.} / d$$

For small angles, $\tan p = p$

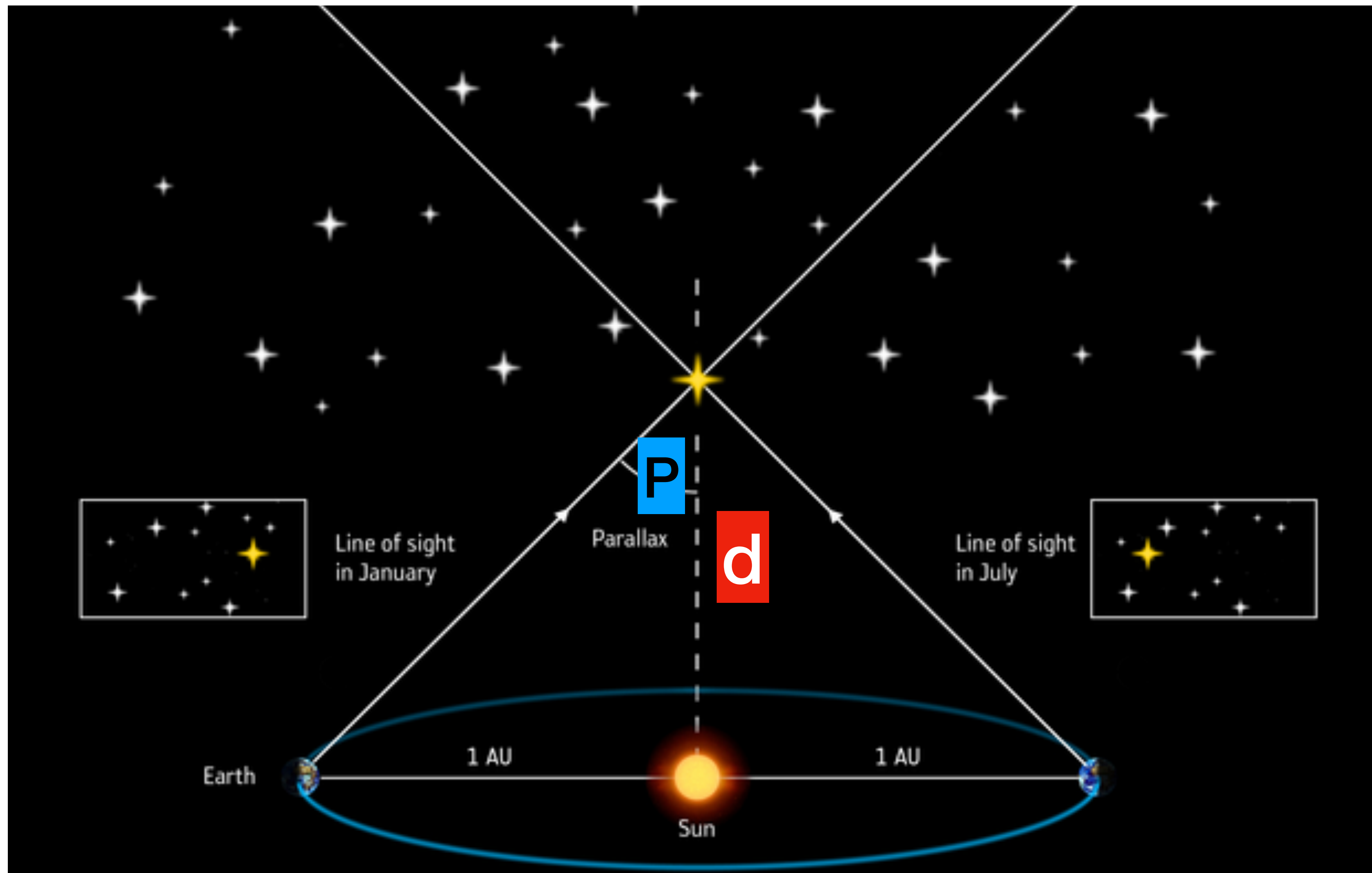
$$P = 1 \text{ A.U.} / d$$

Unit = 1 second
= 1/60 X 1/60 degree

$$D = 1 \text{ A.U.} / p$$

Finding Distances to Nearby Stars

First rung in the Cosmic Ladder (Zero Point) Parallax



$$\tan p = 1 \text{ A.U.} / d$$

For small angles, $\tan p = p$

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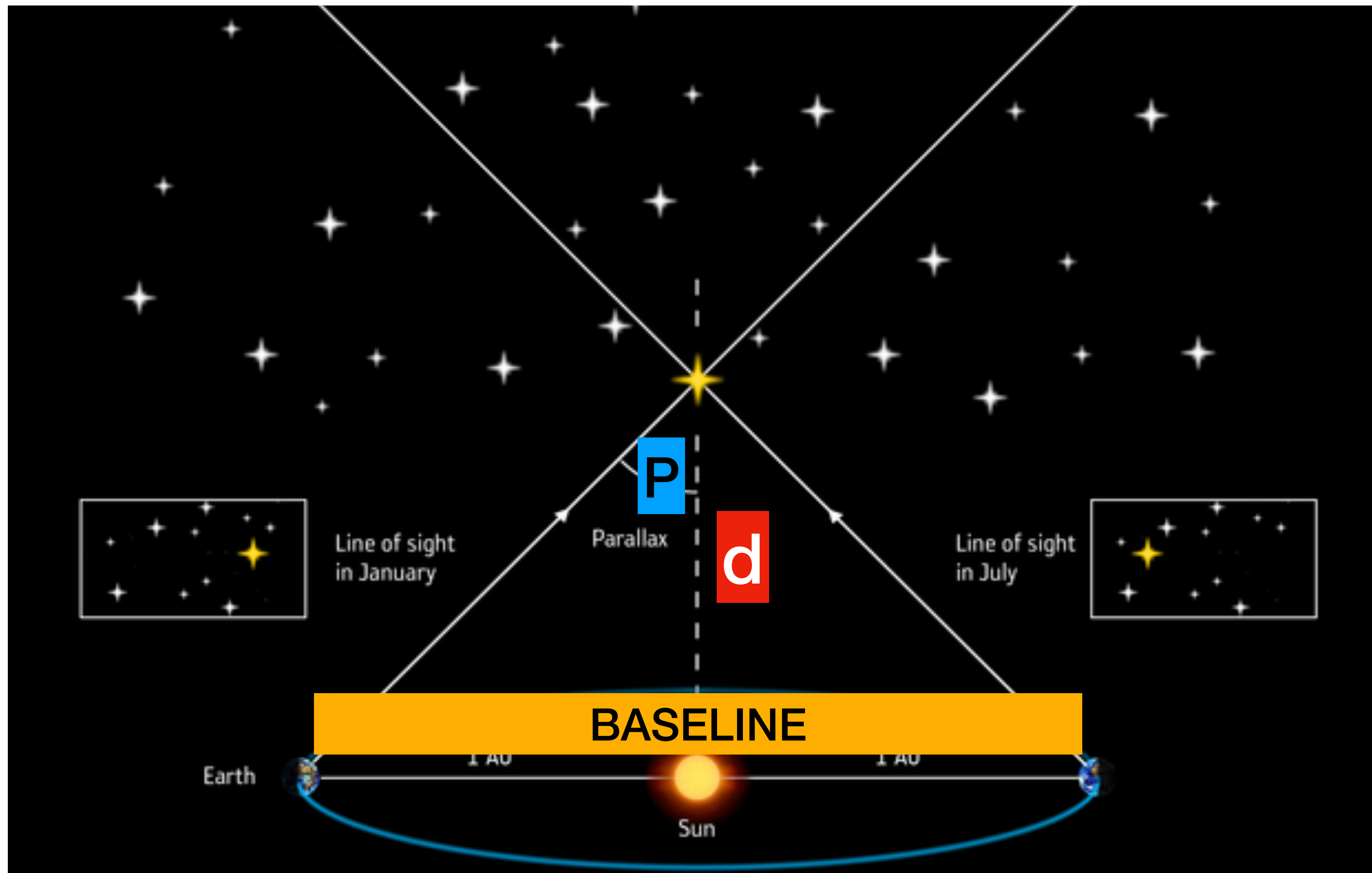
Unit = 1 second
= $1/60 \times 1/60$ degree

$$D = 1 \text{ A.U.} / p$$

If **p** is in seconds
of arc **D** is in units
of parsecs or pc

Finding Distances to Nearby Stars

First rung in the Cosmic Ladder (Zero Point) Parallax



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For small angles, $\tan p = p$

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Unit = 1 second
= $1/60 \times 1/60$ degree

$$D = 1 \text{ A.U.} / p$$

If **p** is in seconds
of arc **D** is in units
of parsecs or pc

Note: Baseline = 2 A.U.

How much is a parsec?

How much is a parsec?

$$\begin{aligned} 1 \text{ parsec (pc)} &= 206\,265 \text{ AU} \\ &= 30.9 \text{ trillion } (10^{12}) \text{ km} \\ &= 3.26 \text{ light years} \end{aligned}$$

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Fun Exercise:

Prove the above using $d = 1 \text{ A.U.} / p$

Hint: Change $p = 1$ sec of arc into degrees and then to radians

Another hint: π radians = 180 degrees

Your answer is in A.U (earth-sun distance)

Note: 1 A.U. = 150 million km

Speed of light is $3.0 \times 10^5 \text{ km/s}$

Limitations of Parallax Method of Finding distances

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* Since $p = 1 / d$, as the distance to a nearby star increases, measuring the parallax angle becomes so small it is impossible to measure accurately

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PROPER MOTION

Limitations of Parallax Method

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*** From ground observations, the atmosphere causes star images to twinkle and blur**

*** Nearby stars also move relative to us, even in a half-year between measurements of the parallax angle: PROPER MOTION**

*** Nearby stars hard to observe against backdrop of bright distant stars, galaxies and interstellar dust**

Parallax Distance Limit: history

Instrument
Used

Measured
P

$$D = 1/p$$

Today's Value: Gaia

Split-lens
Refractor
6 inches
Built by
Fraunhofer

0.314 arc
seconds
61 Cygni

3.18 pc
10.4 Ly

3.5 pc
11.4 LY



Friederich Bessel 1838

Maximum Distance Limit: 10 pc or 33 Ly
Small telescope objective, atmospheric blurring
No photographic plate: just eye and micrometer
device to measure angles (Heliometer)
Could measure distance to only a dozen stars

Parallax Distance Limit: history



Harlow Shapley 1918

Shapley used parallax and Cepheids to get first reasonable measure of size of Milky Way. He incorrectly conjectured Milky Way galaxy was entire Universe

Instrument Used	Measured p	$D = 1/p$	Today's Value: Gaia
Much larger Telescopes Up to 100 inch Hooker 1917	0.37 arc seconds Sirius	2.7 pc 8.8 Ly	2.6 pc 8.6 LY

Maximum Distance Limit: 100 pc or 326 Ly
Larger telescopes, atmospheric blurring
photographic plates
Could measure distance to only a hundred stars

Parallax Distance Limit: history



Willard Boyle 1970's-80's

Canadian Nobel Physics 2009

His work on CCD's or Charge Coupled Devices ushered in more sensitive astrometry at all wavelengths

Instrument Used	Measured p	$D = 1/p$	Today's Value: Gaia
Much larger Telescopes 200 inch more CCD photography	0.75 arc seconds Alpha Centauri	1.3 pc 4.3 Ly	1.3 pc 4.3 LY

Maximum Distance Limit: 500 pc or 1630 Ly
Much Larger telescopes , atmospheric blurring
CCD photography replace photographic plates
Could measure distance to eight thousand stars

Parallax Distance Limit: history



Hipparcos Space Telescope
1989 -1993 NASA
Designed for parallax
distance astrometry

Instrument Used	Measured P	$D = 1/p$	Today's Value: Gaia
Space-based Telescope 11 inch CCD photography	0.00754 arc seconds Polaris Cepheid	132.6 pc 432.5 Ly	136.9 pc 446.5 LY

Maximum Distance Limit: 1 kpc or 3.26 kLy
No atmospheric blurring
CCD photography replace photographic plates
Could measure distance to 2.5 million stars

Parallax Distance Limit: history



Gaia Astrometry Mission
2014-2025
ESA

Instrument Used	Measured p	$D = 1/p$	Today's Value: Gaia
Space-based Telescope Two 1.45 m by .5 m Mirrors 10 m sunscreen CCD photography	0.76807 arc seconds Proxima Centauri	1.302 pc 4.246 Ly	1.302 pc 4.246 Ly

Maximum Distance Limit: 10 kpc or 32.6 kLy
No atmospheric blurring, sensitive near IR (infrared wavelengths)
CCD photography replace photographic plates
Could measure distance to 2 billion stars
More to come in data release 4 (DR4)

Key Ideas of Geometric Parallax to find distances to Stars

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*** direct measure of distance to stars**

Key ideas of Geometric Parallax to find distances to Stars

- * direct measure of distance to stars**

- * Even with the accuracy of the Gaia parallax measurements, we can only measure up to about 30 000 Ly, about a third of the distance across the disk our Milky Way galaxy**

**How can we measure distances
beyond the Milky Way?**

How can we measure distances beyond the Milky Way?

* Need Very Bright “Standard Candles”
with a known Power or Luminosity L
(Watts=Joules/second) [Energy per time]

How can we measure distances beyond the Milky Way?

Apparent Brightness or Flux =
Luminosity / (area of sphere at a distance
away from the star)

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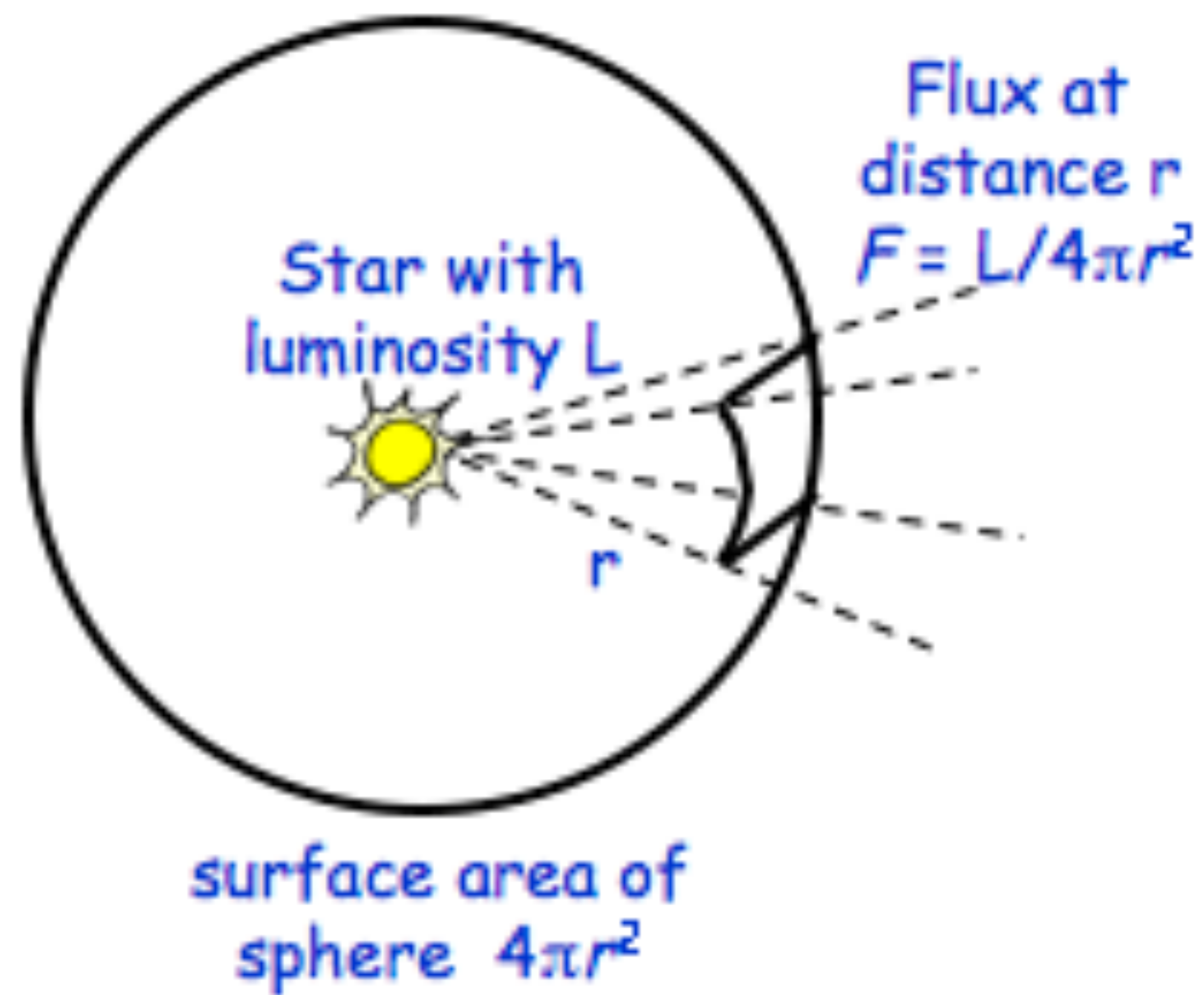
$$F = L / (4 * \pi * D ^ 2)$$

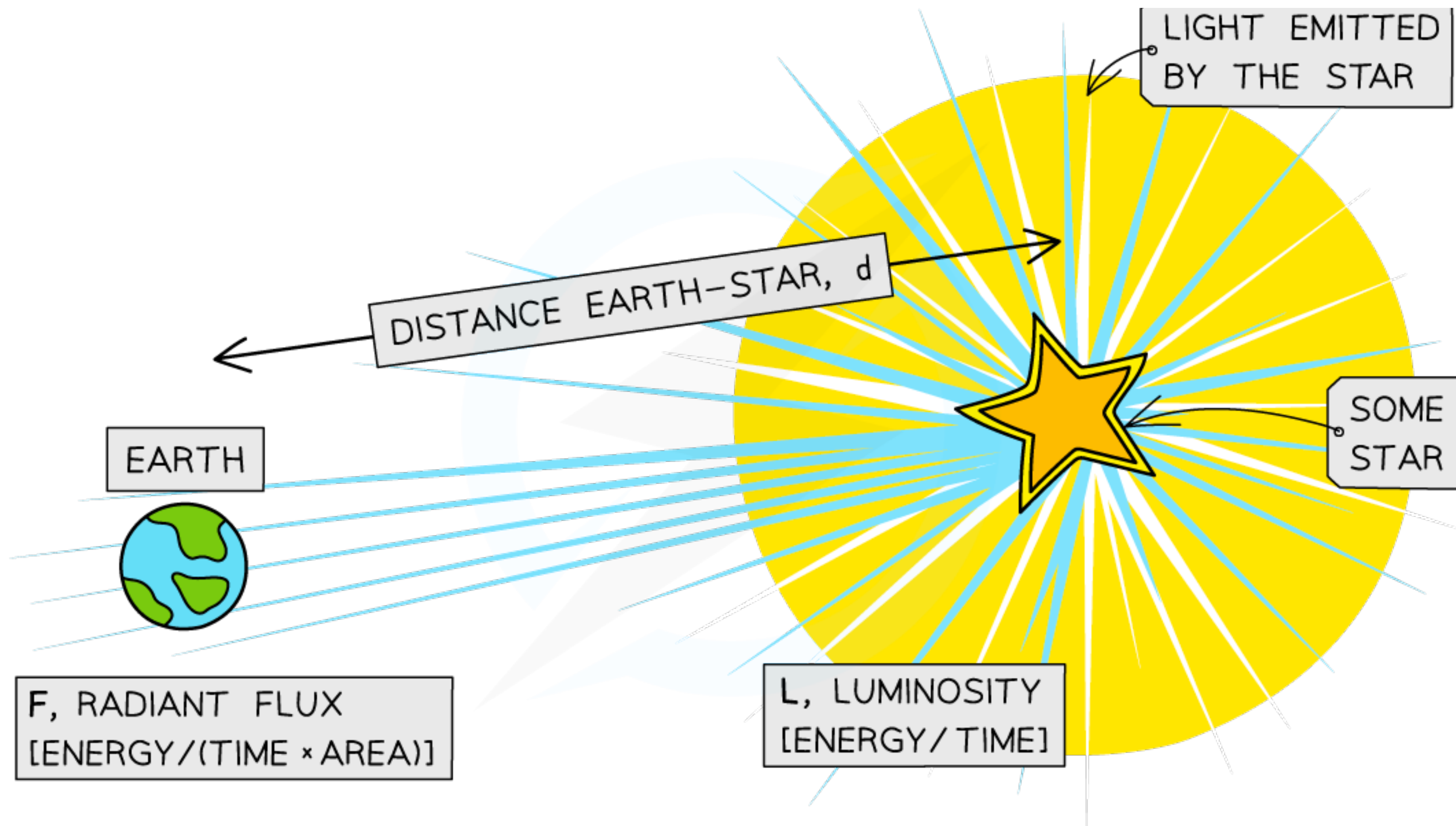
How can we measure distances beyond the Milky Way?

**Apparent Brightness or Flux =
Luminosity / (area of sphere at a distance
away from the star)**

$$F = L / (4 * \pi * D ^ 2) \quad [\text{Units} = \text{W/m}^2]$$

Brightness follows the Inverse square law





Ratio of energy in Joules striking a surface per second per square meter is constant all over the sphere

Calculating the Sun's distance using flux

Calculating the Sun's distance using flux

$L \text{ of the sun} = 3.8 \times 10^{26} \text{ Watts}$

Calculating the Sun's distance using flux

L of the sun = 3.8×10^{26} Watts

Suppose we measure the energy of the sun's
light hitting 1 square meter of solar panel each
second

Calculating the Sun's distance using flux

L of the sun = 3.8×10^{26} Watts

Suppose we measure the energy of the sun's light
hitting 1 square meter of solar panel each second

= Flux from sun

= Solar constant

= 1400 W /m^2

Calculating the Sun's distance using flux

$$F = L / (4 * \text{Pi} * d^2)$$

Calculating the Sun's distance using flux

$$F = L / (4 * \pi * D^2)$$

Solve for D

$$D = \text{square root} (L / 4 * \pi * F)$$

Calculating the Sun's distance using flux

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$$D = \text{square root} (L / 4 * \pi * F)$$

$$D = \text{square root} (3.8 \times 10^{26} / (4 \times \pi \times 1400))$$

Calculating the Sun's distance using flux

$$F = L / (4 * \pi * D^2)$$

Solve for D

$$D = \text{square root} (L / (4 * \pi * F))$$

$$\begin{aligned} D &= \text{square root} (3.8 \times 10^{26} / (4 \times \pi \times 1400)) \\ &= 1.5 \times 10^{11} \text{ m or 1 A. U.} \end{aligned}$$

How can we measure distances to other galaxies?

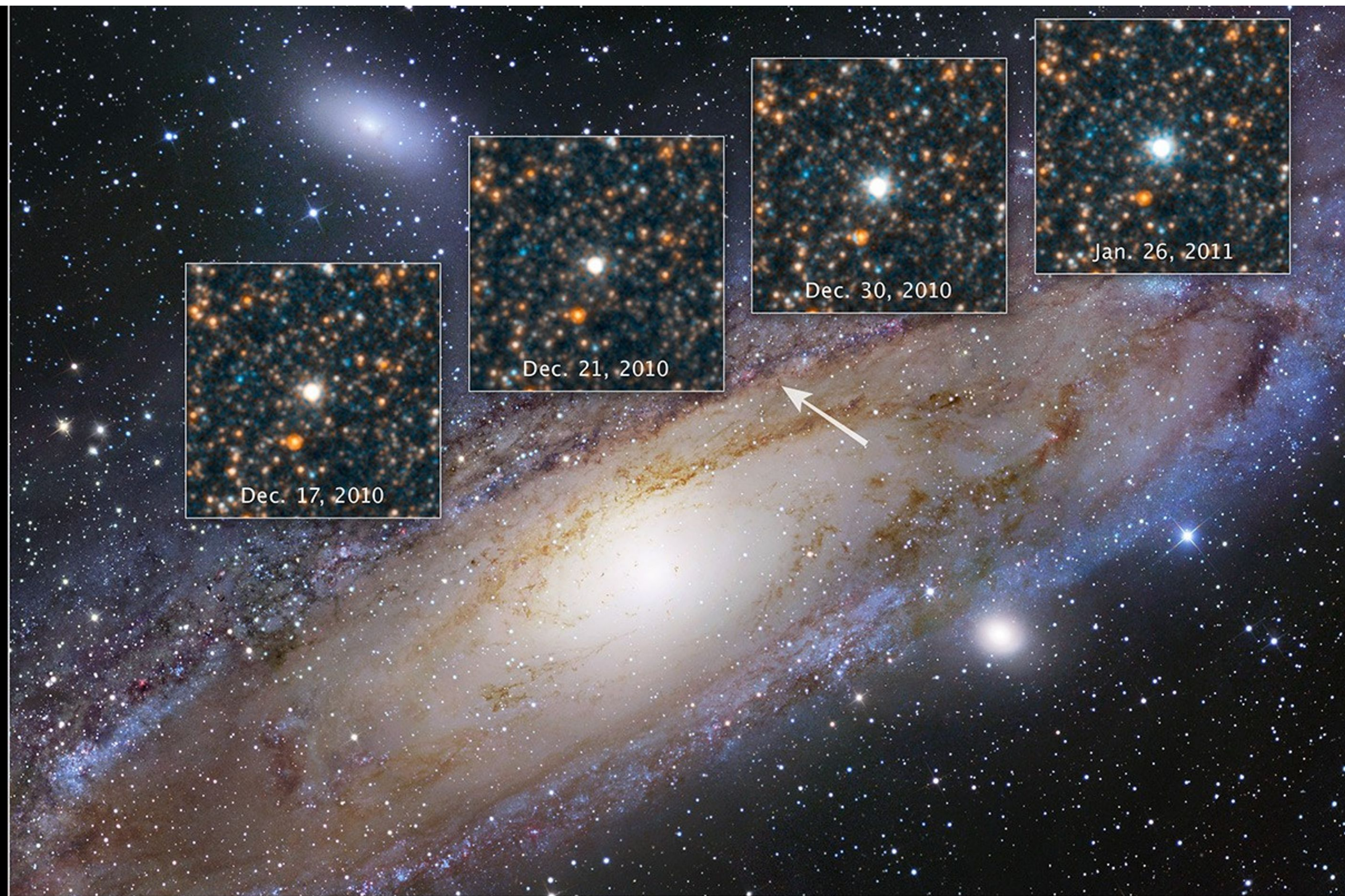
How can we measure distances to other galaxies?



Henrietta Leavitt 1912

As a “computer” of Harvard Observatory, Leavitt examined **Cepheid** variable stars in a small satellite galaxy of the our own Milky Way Galaxy called the **Small Magellanic Cloud (SMC)** .

These dying stars pulsate both in brightness, diameter, and temperature.

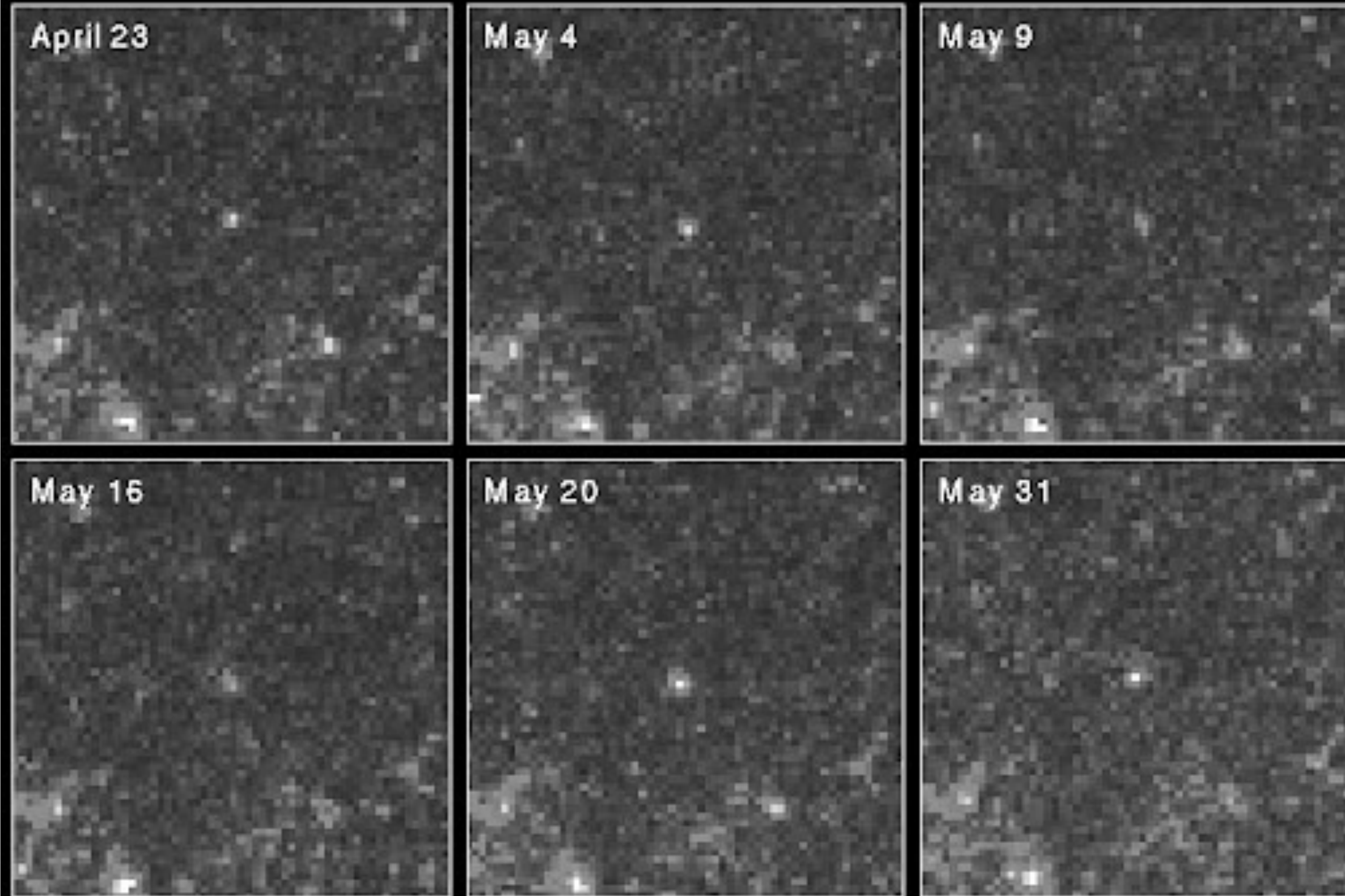


Cepheid Variable Star V1 in M31
Hubble Space Telescope ■ WFC3/UVIS

V1, the famous Cepheid star that Hubble used to find the distance to M31.
The universe was bigger than the our Milky Way Galaxy! (1923)

Cepheid Variable Star in Galaxy M100

HST-WFPC2



A more recent photo of a Cepheid variable 51 million light years away in M100. Between 1908 and 1912, Henrietta Leavitt was the first to study and realize their importance to determine distances beyond our Galaxy!

How can we measure distances to other galaxies?



Leavitt was the first to determine several important attributes of Cepheids

Henrietta Leavitt 1912

How can we measure distances to other galaxies?



Henrietta Leavitt 1912

Leavitt was the first to determine several important attributes of Cepheids

*** Cepheids were extremely bright, luminous stars that were found not only in SMC, but in our own galaxy and other “nebula” as well**

How can we measure distances to other galaxies?

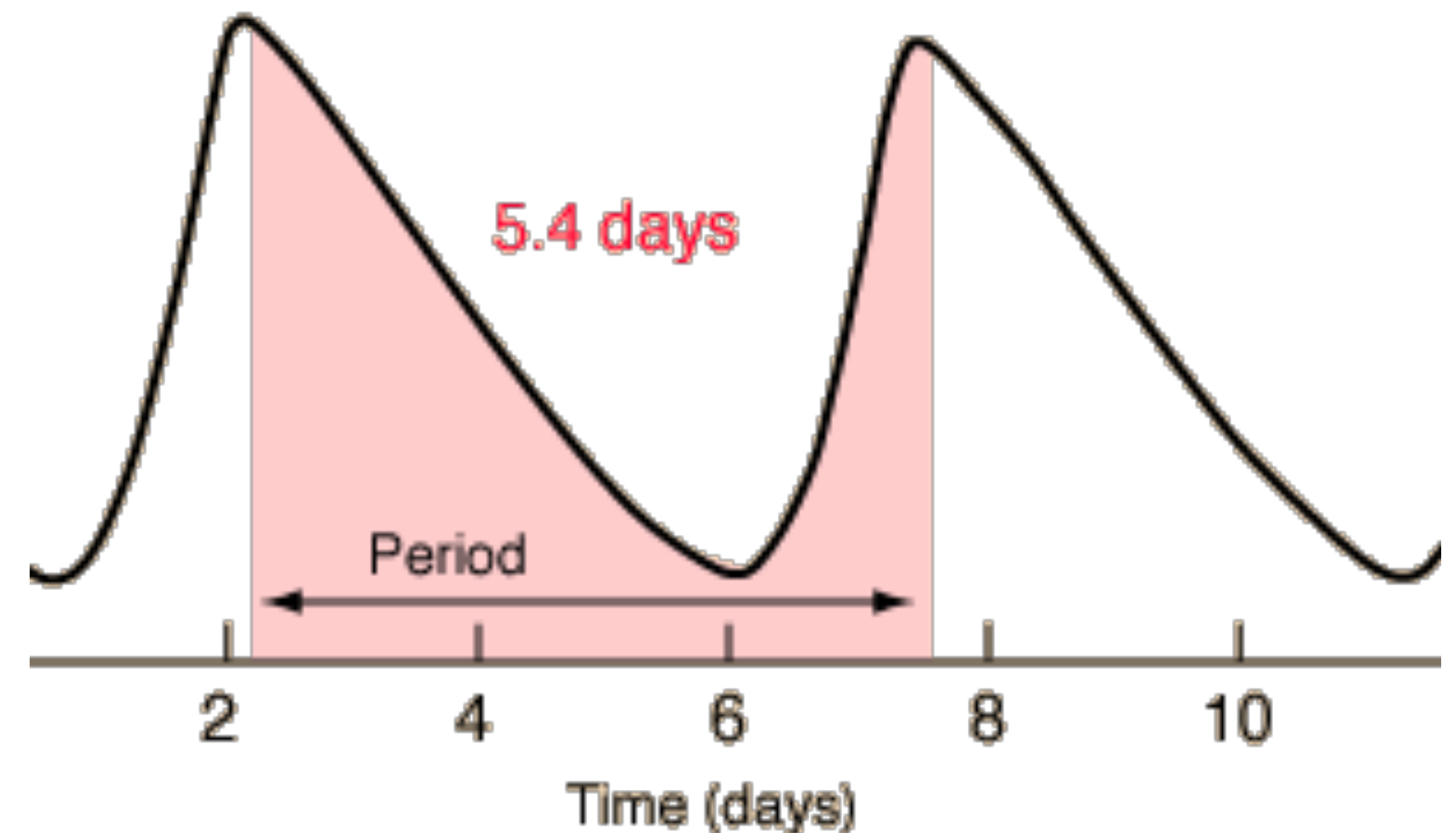


Henrietta Leavitt 1912

Leavitt was the first to determine several important attributes of Cepheids

*** All Cepheids had a definite period of pulsation**

Brightness variation of δ -Cephei
3.6 - 4.3 Magnitude



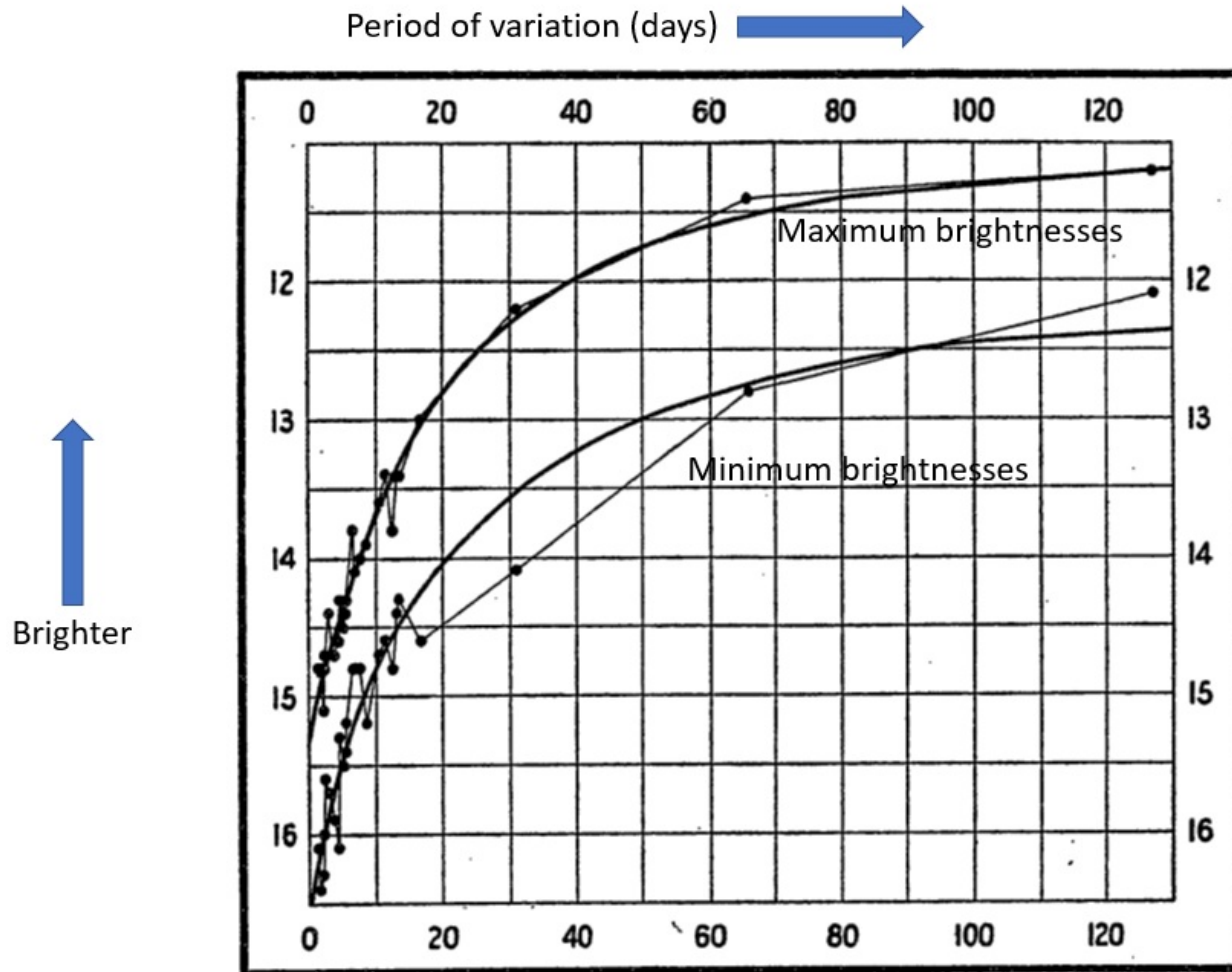
How can we measure distances to other galaxies?



Henrietta Leavitt 1912

Leavitt was the first to determine several important attributes of Cepheids

*** As the period of the Cepheid increased, the apparent brightness or “flux” increased in a very predictable mathematical way as well**



1912 graph by Leavitt of Cepheid stars “Flux” or
“Apparent Brightness” in SMC vs period

How can we measure distances to other galaxies?

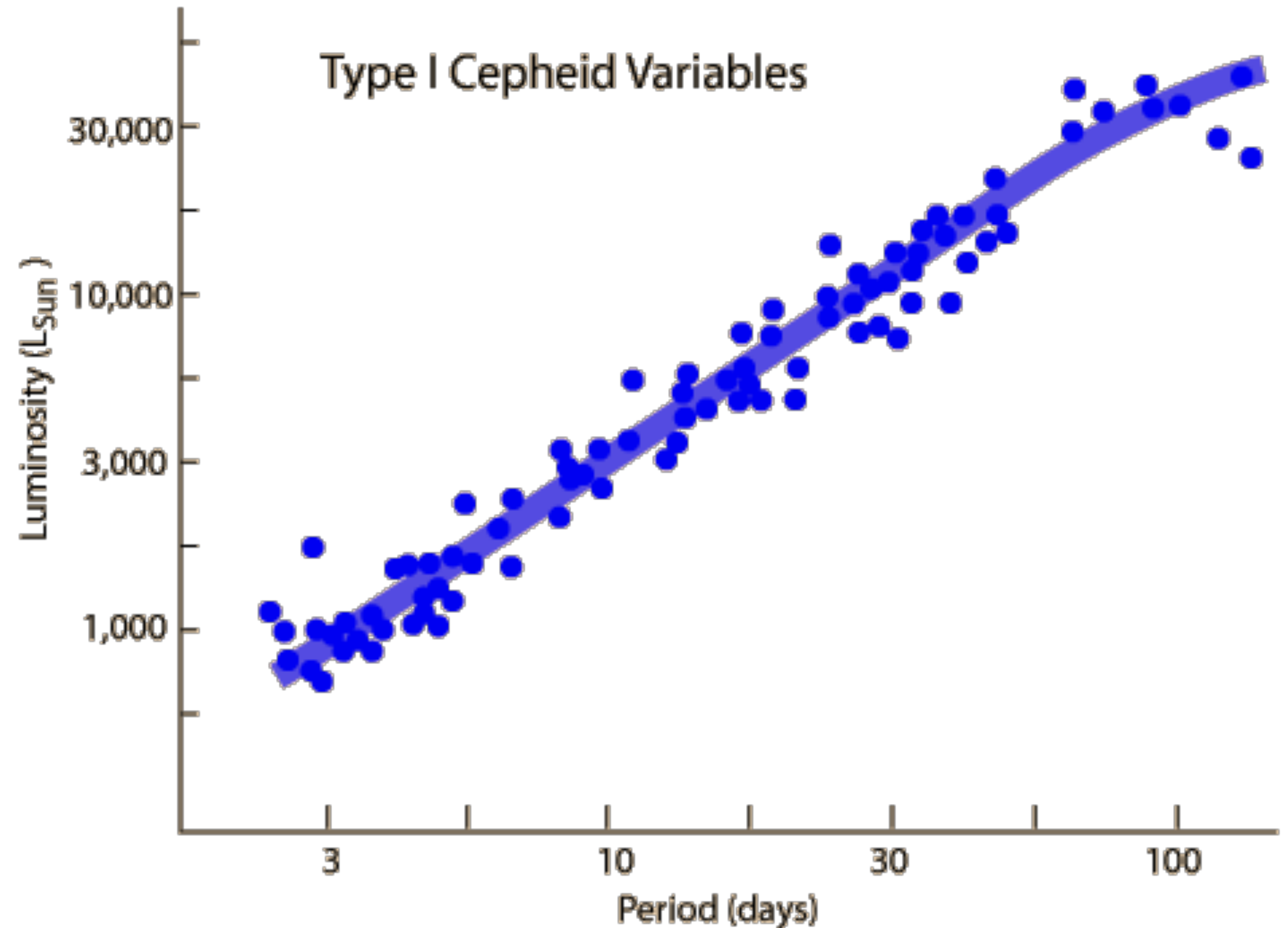


Henrietta Leavitt 1912

Leavitt was the first to determine several important attributes of Cepheids

- * **Since all the Cepheids in the SMC were the same distance away, Leavitt concluded that their flux or brightness was directly proportional to each star's Luminosity by $F = L / (4 * \pi * d^2)$**

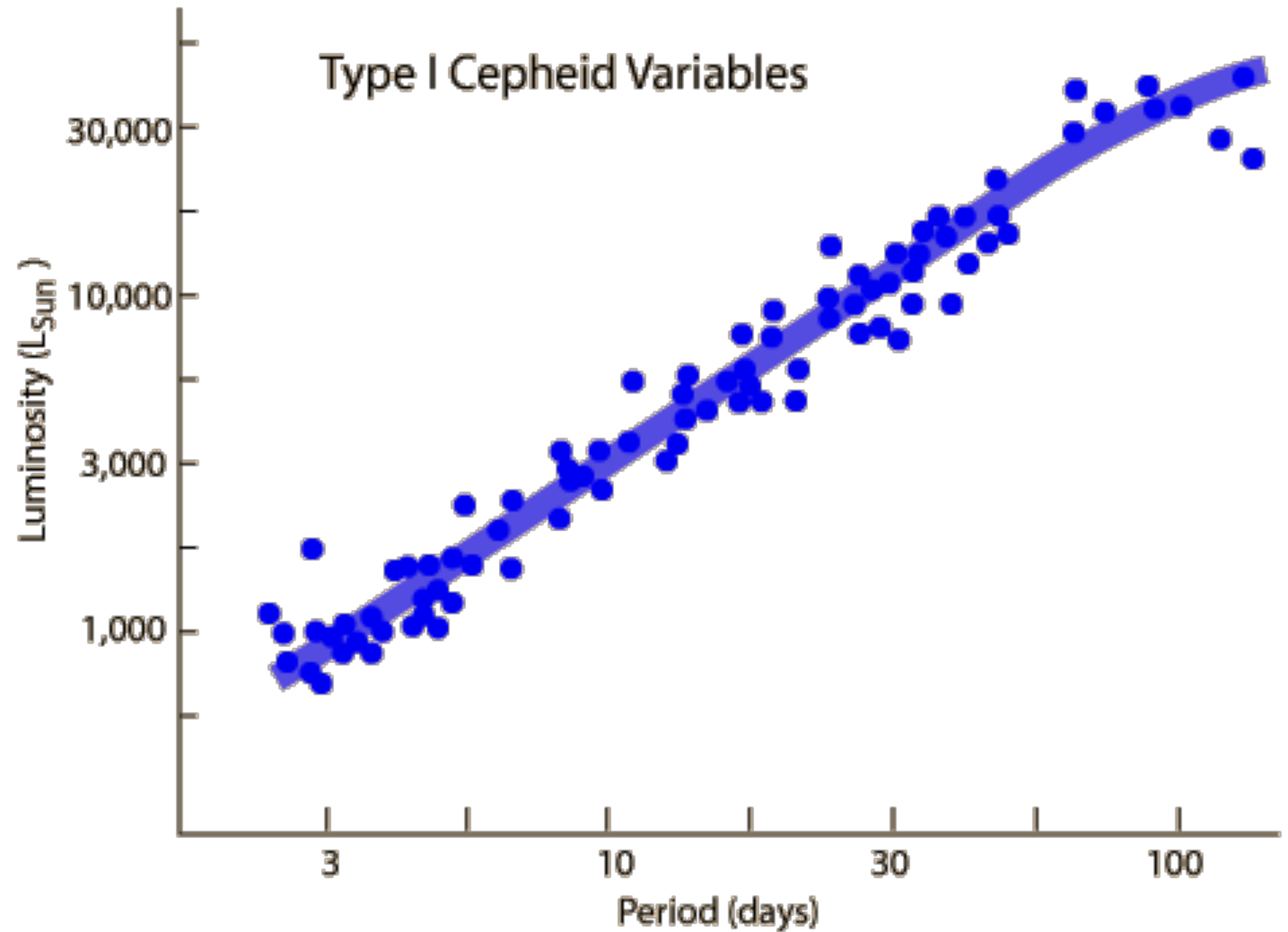
Leavitt asserted the apparent brightness or “flux” on the vertical axis could be replaced with the actual luminosity of the Cepheid.



“Leavitt” **Period - Luminosity Relation** for
Cepheids

This was done by measuring distances to nearby Cepheid stars like Polaris and delta Cephei in our own Galaxy by parallax. Using the inverse square formula, luminosity can be easily calculated.

$$F = L / (4 * \pi * d ^ 2)$$



“Leavitt” **Period - Luminosity Relation** for Cepheids

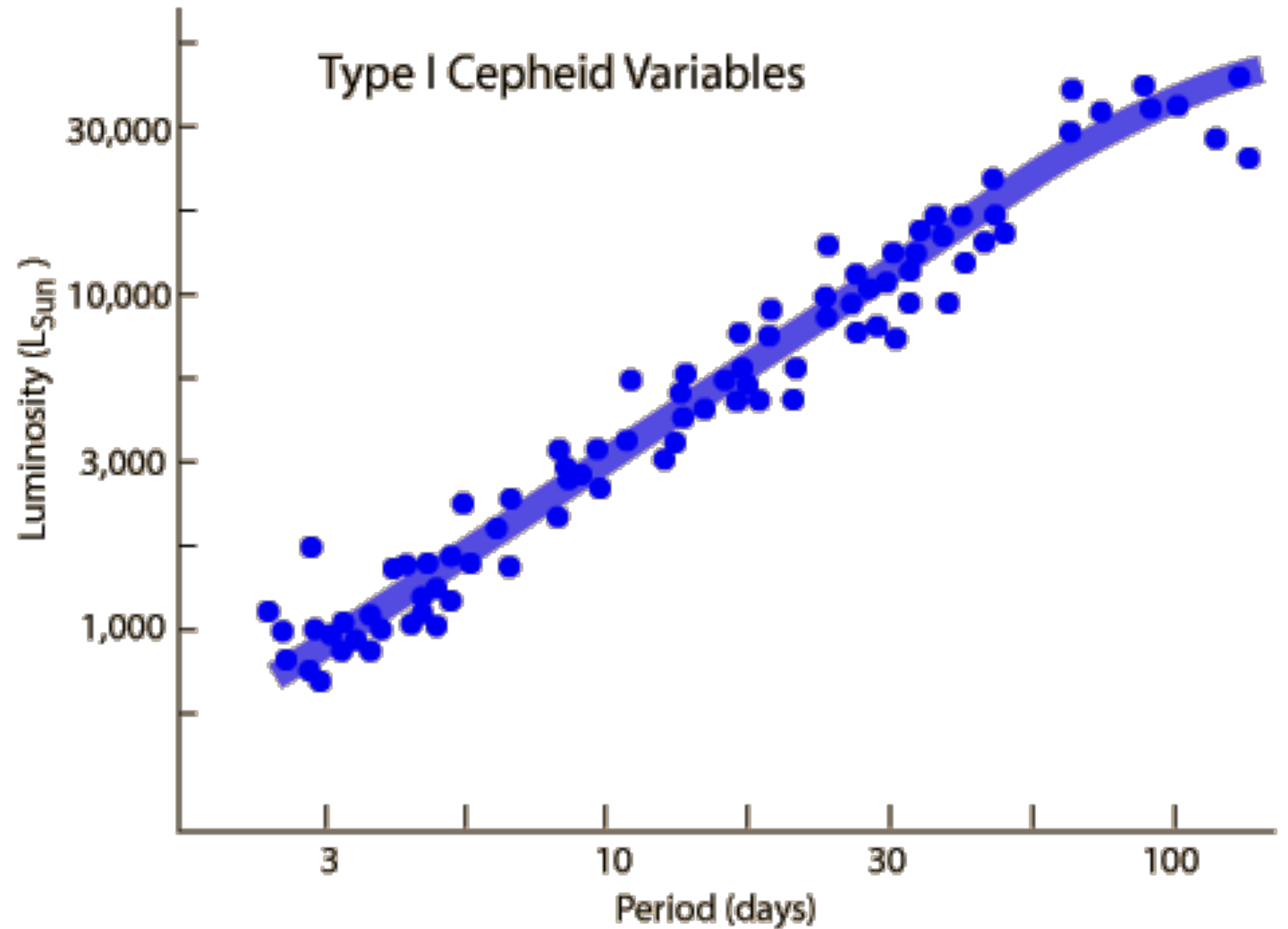
Inverse square law

$$F = L / (4 * \pi * d ^ 2)$$

This is called “calibrating”
the Cepheids to a
luminosity.

Harlow did this to help
make a graph similar to
the one seen.

Hubble measured the
period of
his famous “V” star. He
then just read the
luminosity from the graph,
plugged it in to the inverse
square law along with the
flux (brightness), and
determined the distance to
M31!



“Leavitt” **Period - Luminosity Relation** for Cepheids

Key notes about Leavitt's Law of Cepheids

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Cepheids are thousands of times more luminous than our sun, so they can be seen in galaxies up to 100 million light years away!

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Because each Cepheid has a measurable period, its luminosity can be determined by Leavitt's period - luminosity law. From the luminosity, its distance and the distance to the galaxy it is in can be determined from the inverse square formula shown!

Key notes about Leavitt's Law of Cepheids

Because each Cepheid has a constant max/min/average Luminosity, they are called **STANDARD CANDLES** in astronomy. Distances determined in this way are called **LUMINOSITY DISTANCES!** Nearby Cepheids distance are calculated using the **first rung** in the cosmic distance ladder, **PARALLAX**, as a calibration. **CEPHEIDS** are the **second rung** in the ladder to determine greater distances.

More Notes on luminosity distances

More Notes on luminosity distances

There are actually two populations of Cepheids.

“Classical” or Type One Cepheids are more massive metal rich stars, typically 4 to 20 times the mass of the sun, but they are in their death throes of pulsation. Type Two Cepheids are older metal-poor stars sometimes found in globular clusters with masses at about one-half a solar mass. Both types are very luminous, but have a different period-luminosity relationship.

More Notes on luminosity distances



Walter Baade
circa 1940's

Before the late 1940's and the discovery of the types of Cepheids by **Walter Baade**, all Cepheids were thought of as being the same type. This caused Harlow, Hubble, and other astronomers to make errors of scale on distance and measurements of the "Hubble constant". However, their methods had a sound scientific basis.

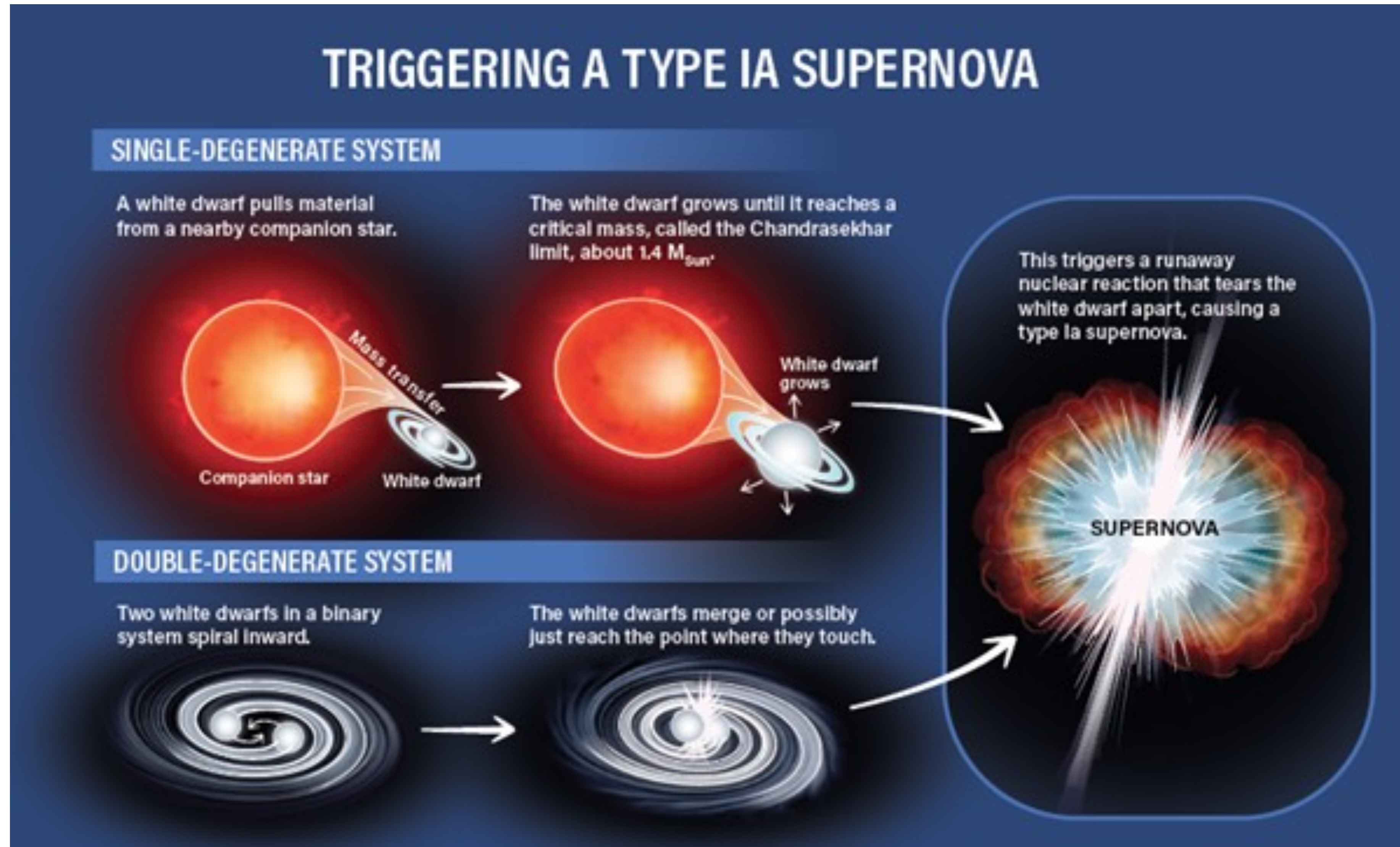
Luminosity distances from 1a Supernovas

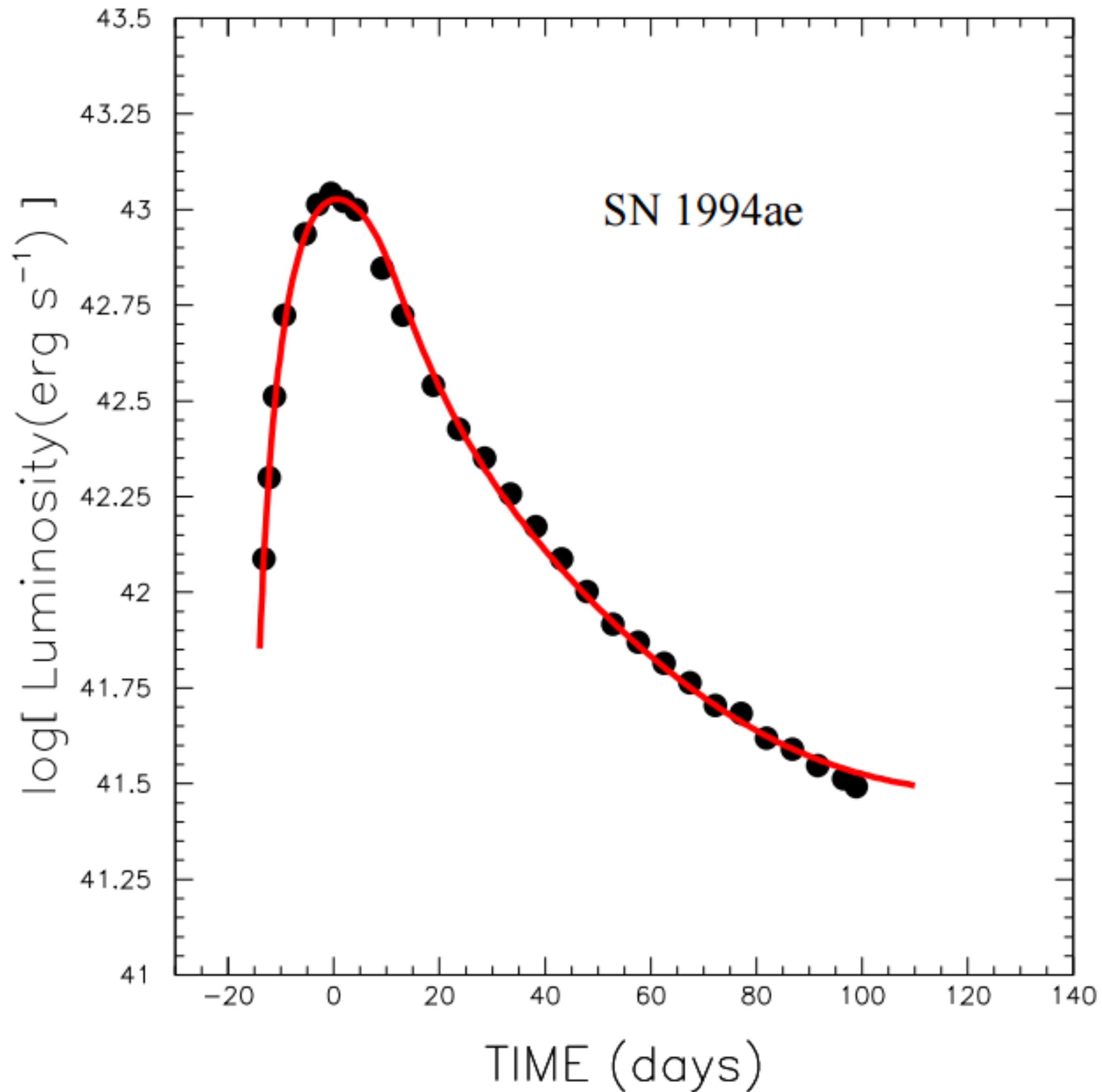


A **Type 1a supernova** explodes in the pinwheel galaxy! These supernovae explode the same mass every time (1.4 X solar mass) and have an easily calculable luminosity. From the apparent brightness of the supernova, the distance to the galaxy can be determined!

Type 1a supernova are even brighter standard candles to find luminosity distances. They are 250 000 times more luminous than Cepheids and equal to 5 billion suns at their peak! Unlike core-collapse supernovas, they completely obliterate the white dwarf in a runaway nuclear reaction.

Luminosity distances from 1a Supernovas





All type 1a supernova are due to 1.4 solar Masses exploding, so the peak luminosity is fairly constant at about **5 billion suns** (Hipparchus absolute magnitude = -19.3)

There are variations in the peak but astronomers know how to correct any changes in the peak luminosity by looking at the shape of the curve: **Phillips Relation**

Luminosity distances from 1a Supernovas



Adam Riess

Nobel Prize 2011

In 1998 Riess used 1a supernovas to determine luminosity distances of up to 12 GLy away!

These supernovas were 10 - 15 % fainter than expected in a strictly matter dominated universe.

An expanding universe that switched from slowing down to speeding up about 5 billion years ago due to “dark” energy “explained” the results! (Lambda-CDM Model)

Type 1a supernova are the third rung in the cosmic distance ladder.

Distances to 1a supernova are “calibrated” with Cepheids at luminosity distances from 60 to 250 MLy. In 2021, Adam Riess and his SHoES team used 1a supernova in measuring luminosity distances from 30 MLy to 300 MLy to find Hubble’s constant now.

$$H_0 = (da/dt) / a \text{ (fractional expansion rate)}$$

Recession Speed of Galaxies and “Redshift”

Recession Speed of Galaxies and “Redshift”

In 1929, Hubble produced evidence for an expanding universe, and ever since then astrophysicists have been trying to find how the size of the observable universe changes over time by finding $H(t)$, the Hubble-Lemaitre Parameter. In order to find H_0 the Hubble-Lemaitre parameter now, astronomers needed to find the distance to far-away galaxies and how fast they were moving away from “us”.
(Centre of Milky Way Galaxy)

Recession Speed of Galaxies and “Redshift”

Hubble's observations in 1929 offered an
equation to find H_0

$$H_0 = V_R / D_P$$

Recession Speed of Galaxies and “Redshift”

$$H_0 = V_R / D_P$$

Hubble's Law

Recession Speed of Galaxies and “Redshift”

$$H_0 = V_R / D_P$$

Hubble's Law

D_P = the tape measure distance or the distance to the galaxy now, called **proper distance** in Mpc (3.26 MLy)

This distance is measured from the centre of the Milky Way as our frame of reference.

Recession Speed of Galaxies and “Redshift”

$$H_0 = V_R / D_P$$

D_P = the tape measure distance or the distance to the galaxy now called **proper distance** in Mpc (3.26 MLy)

Note: For luminosity distances less than 300 MLy, proper distance and luminosity distance are within 3%

Recession Speed of Galaxies and “Redshift”

$$H_0 = V_R / D_P$$

V_R = relative speed away from the centre of
the Milky Way due to the expansion of
space/scale factor in **km/s**

Recession Speed of Galaxies and “Redshift”

$$H_0 = V_R / D_P$$

V_R = relative speed away from the centre of the Milky Way due to the expansion of space/scale factor in **km/s**

NOT

V_P or the local Peculiar Velocity of a galaxy through space due to local gravitational effects

Recession Speed of Galaxies and “Redshift”

$$H_0 = V_R / D_P$$

For example, Andromeda Galaxy M31 has a peculiar speed through space at $V_P = 165$ km/s toward “us” but space is expanding at $V_R =$ recession speed of 55 km/s away from “us” (Milky Way centre)

So...

M31 is still moving toward us at 110 km/s

Recession Speed of Galaxies and “Redshift”

$$H_0 = V_R / D_P$$

A D_P gets bigger, V_R must get bigger too to keep H_0 constant. This means V_R can get even larger than the speed of light c for far away galaxies. V_P of galaxies through space due to gravitational effects stay less than 1000 km/s no matter what. At large scales and distances, for D_P greater than 300 MLY, all galaxies are moving away from us.

Recession Speed of Galaxies and “Redshift”

$$H_0 = V_R / D_P$$

**We know how distances are measured, but what
about recession speeds V_R ?**

Vesto Slipher, Recession Speeds and “Redshift”



Vesto Slipher (1875 -1969)

Vesto Slipher, Recession Speeds and “Redshift”



**Worked at Lowell
Observatory in
Flagstaff Arizona
from 1901 until 1954**

Vesto Slipher (1875 -1969)

Vesto Slipher, Recession Speeds and “Redshift”



Vesto Slipher (1875 -1969)

First to measure the radial velocities of spiral nebulae and galaxies using a spectrograph attached to a 24 inch refractor

Vesto Slipher, Recession Speeds and “Redshift”



Vesto Slipher (1875 -1969)

In 1912, first to measure the radial velocity of M31, the Andromeda Galaxy at 300 km/s towards us relative to the sun and 110 km/s relative to the centre of the Milky Way)

Vesto Slipher, Recession Speeds and “Redshift”



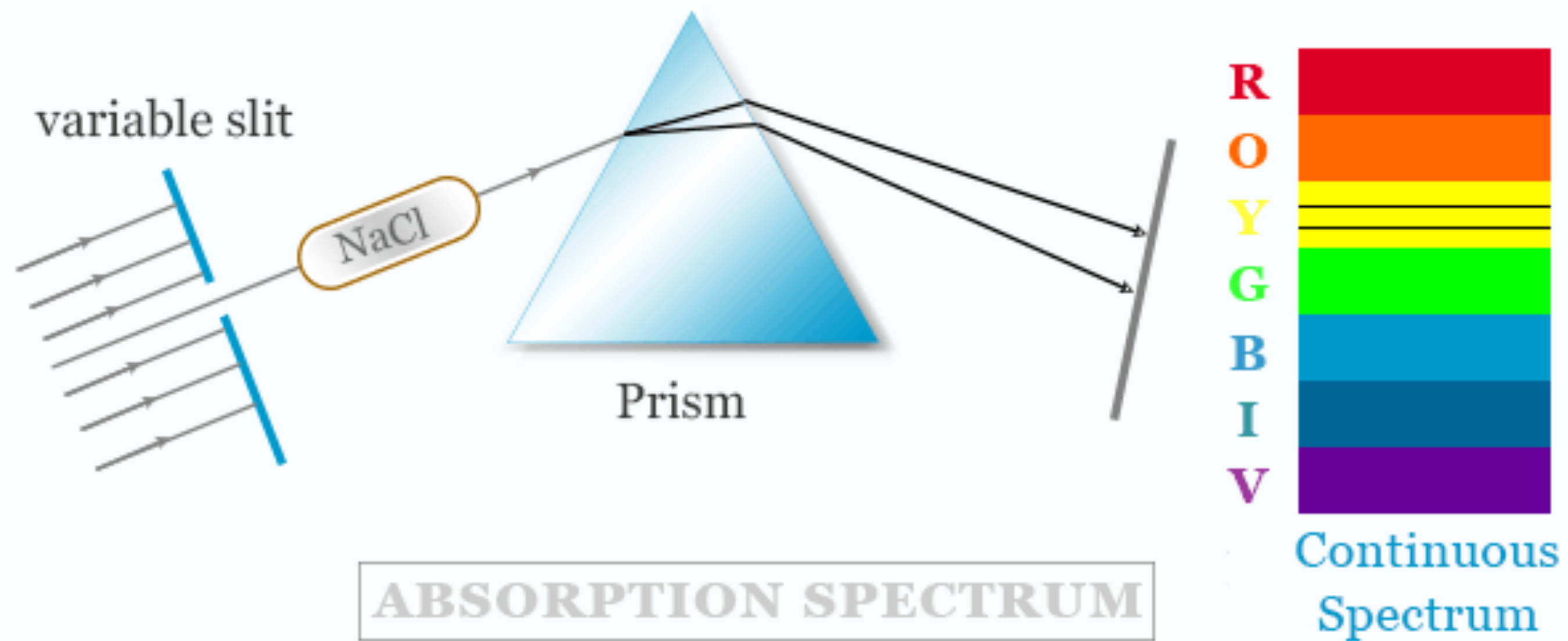
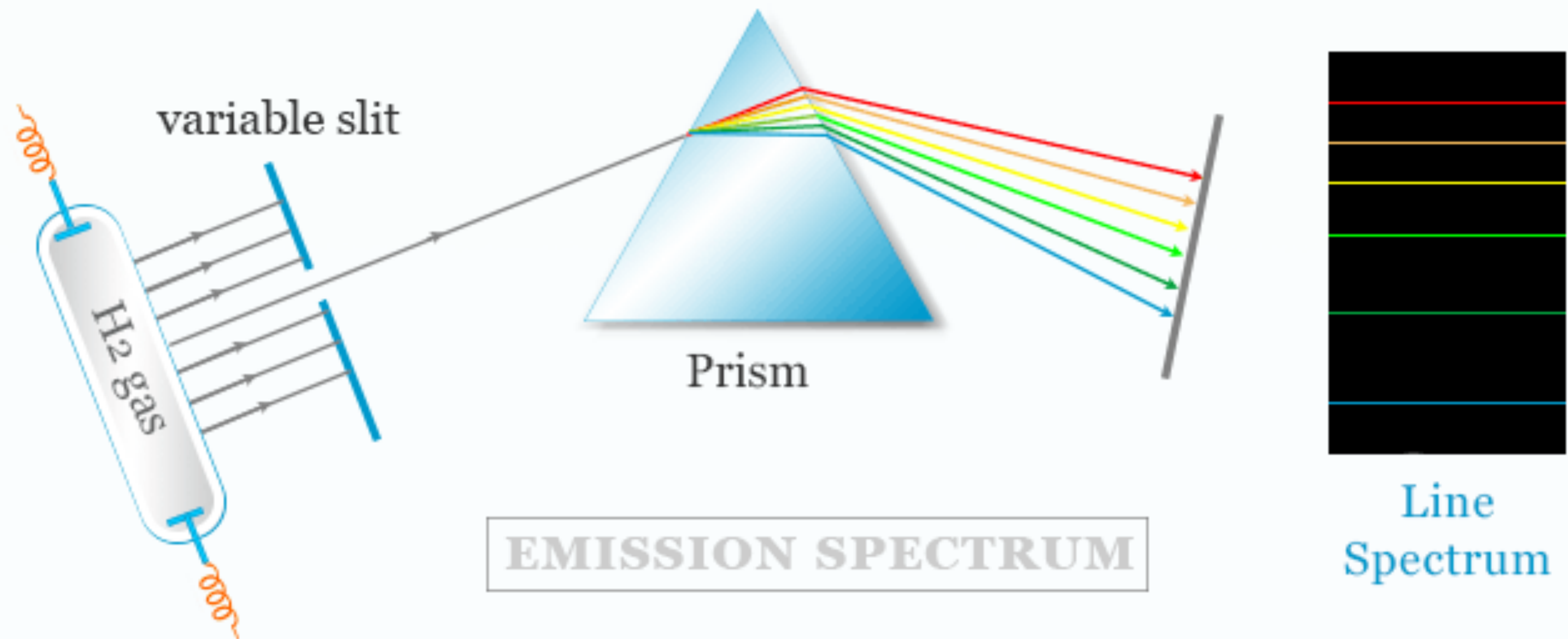
Vesto Slipher (1875 -1969)

In 1917, Slipher found that 21 out of 25 spiral galaxies had radial velocities away from “us” ! His data helped Lemaitre and Hubble assert Hubble’s Expansion Law (1927 & 1929)

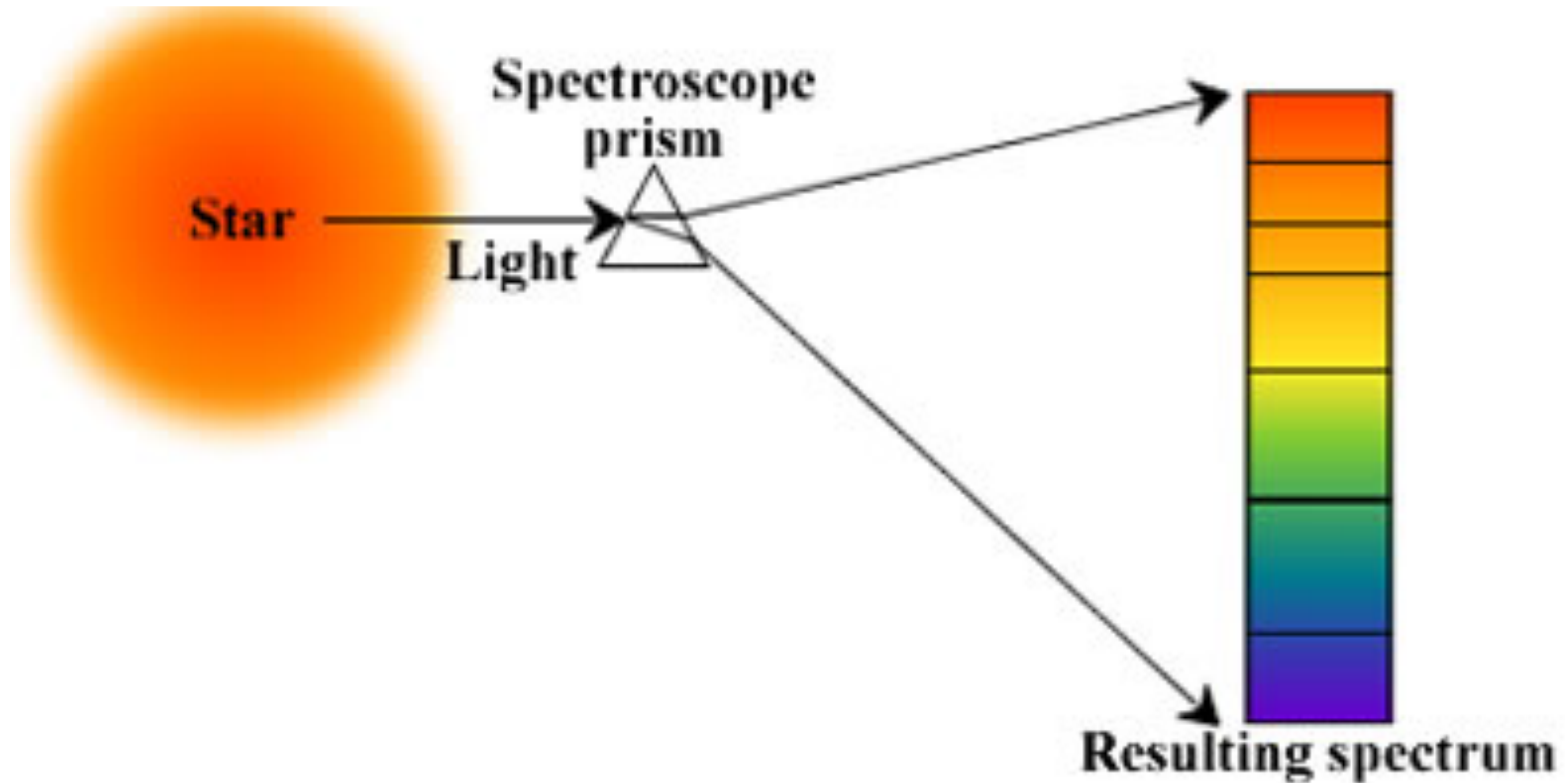
$$H_0 = V_R / D_P$$

**How did Slipher find the radial
velocities of galaxies?**

SPECTROSCOPY



Three types of spectra



Applying Spectroscopy to Astronomy: **Slipher**
looked at absorption lines in starlight

Two Ways Absorption Lines “Shift” position

Two Ways Absorption Lines “Shift” position

A line on a star at rest to “us” will have a definite wavelength λ in nm (billionth of a meter) called λ_e (emitted wavelength) and can be measured on a spectrometer or spectrograph.

Two Ways Absorption Lines “Shift” position

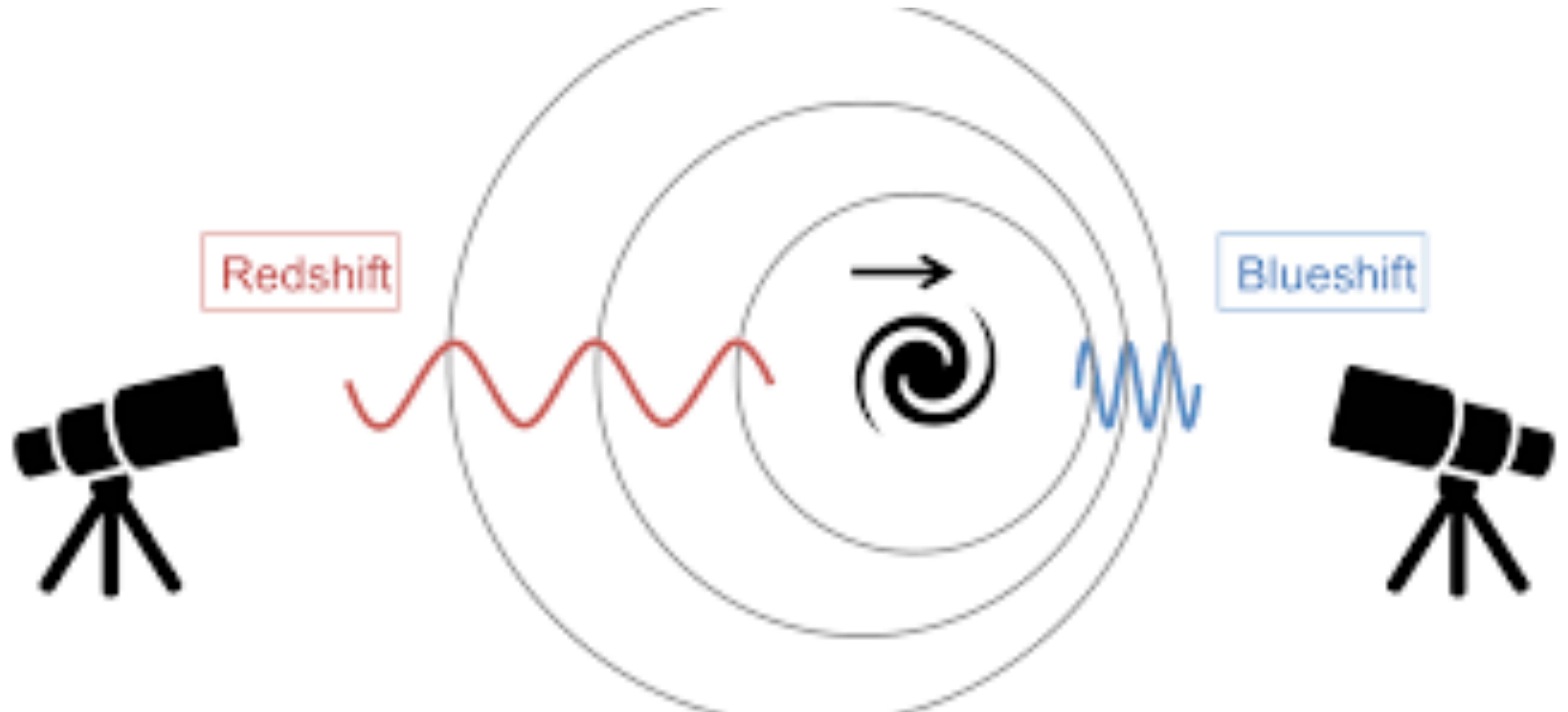
If the star is moving **away** from “us”, the light’s wavelength gets **longer** and is observed by us as **λ_o (observed wavelength)**. The line shifts position on the spectrum toward the longer wavelengths like red and is said to be **“redshifted”**. The wavelength does not necessarily correspond to a “red” colour! In fact, the line could correspond to a wavelength so long it is “infrared” and no longer visible to us, but its position on the spectrum and its wavelength can still be measured.

$$Z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

Redshift Defined: It is much easier to measure redshift of starlight off a spectrometer or spectrograph than to measure distances. The symbol for redshift is **Z**

Two Ways Absorption Lines “Shift” position

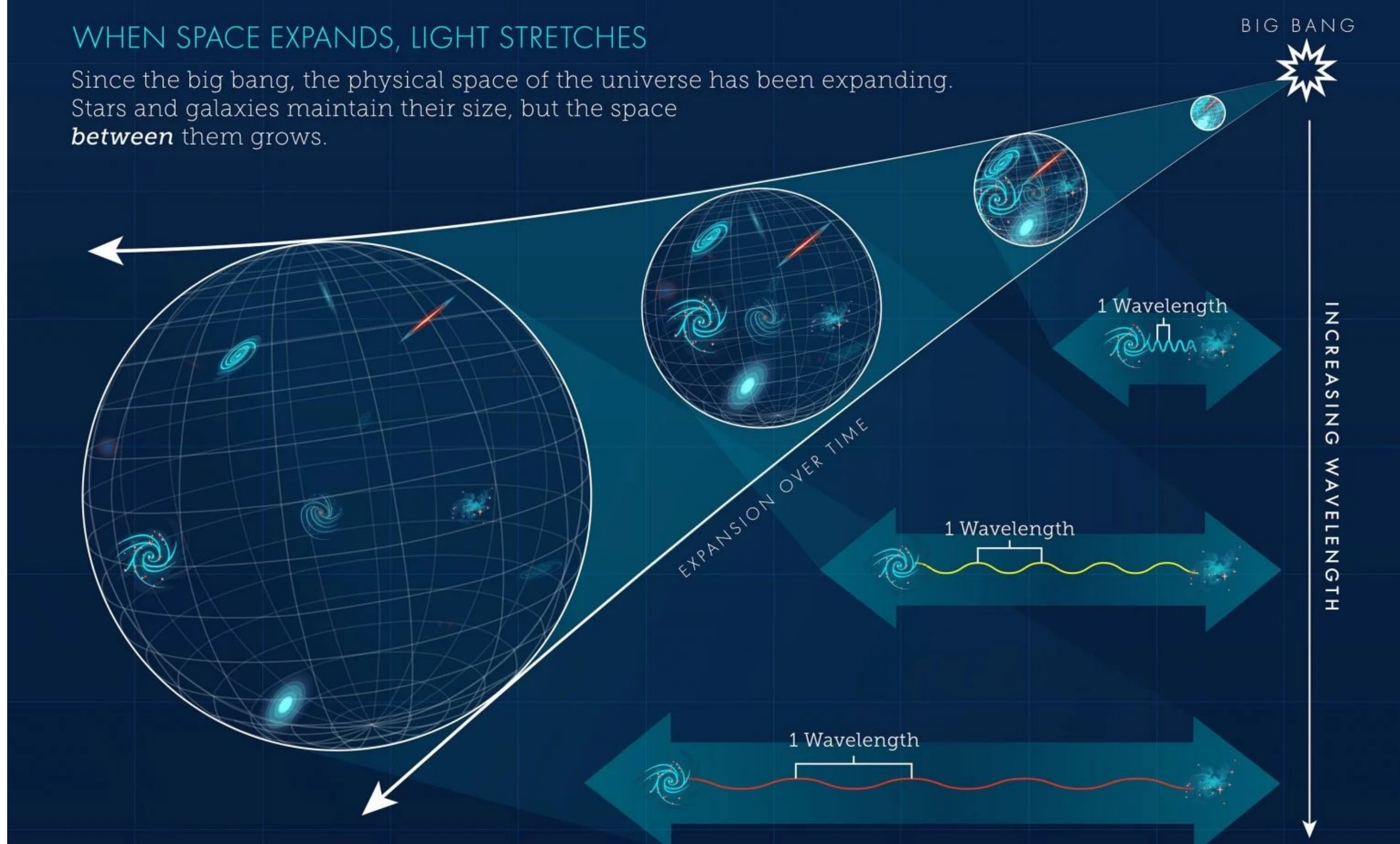
If the star is moving **toward** “us”, the light’s wavelength gets **shorter** and is observed by us as **λ_o (observed wavelength)**. The line shifts position on the spectrum toward the shorter wavelengths like blue and is said to be **“blueshifted”**. The wavelength does not necessarily correspond to a “blue” colour! In fact, the line could correspond to a wavelength so short it is “ultraviolet” and no longer visible to us, but its position on the spectrum and its wavelength can still be measured. Z will be negative!



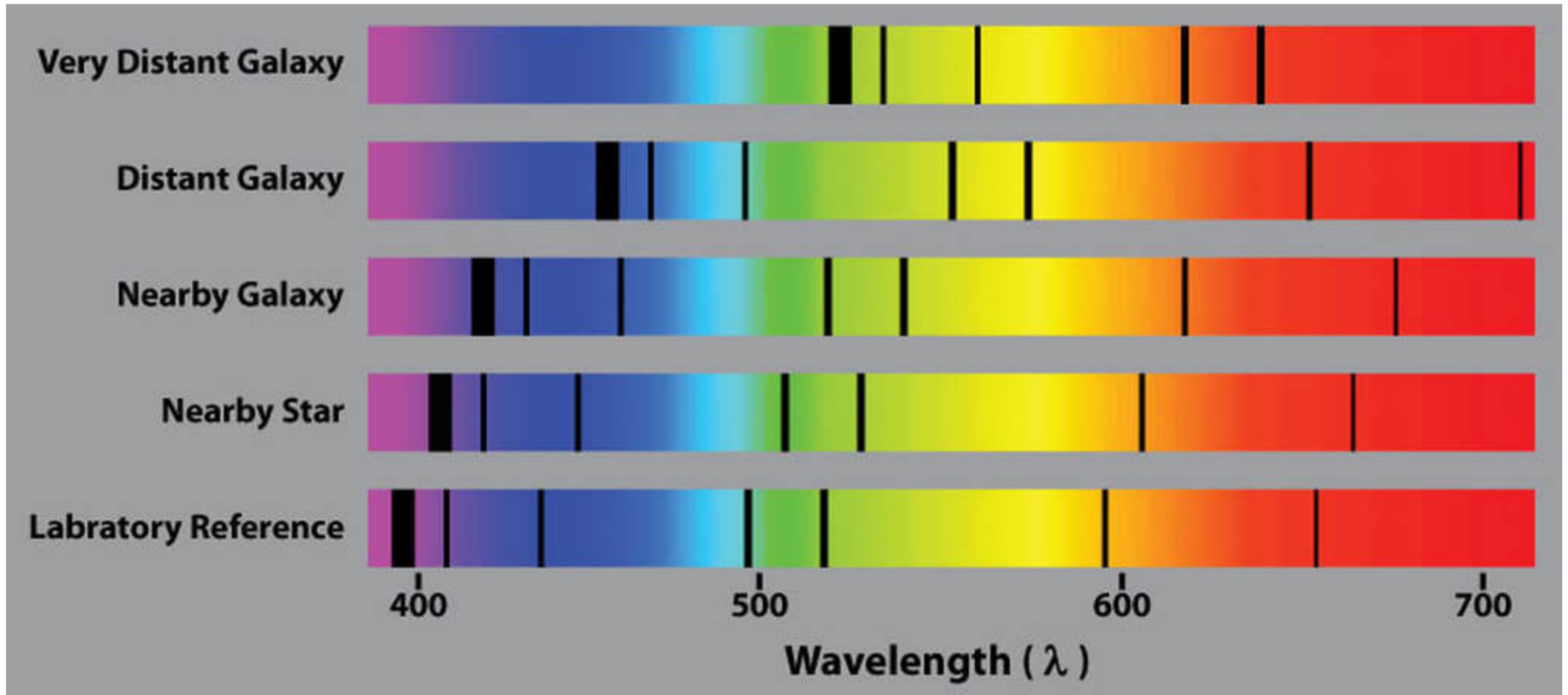
When a galaxy moves through space due to local gravitational effects it is “Doppler-shifted”. The wavelength of the light can get longer or shorter! This kind of effect corresponds to peculiar velocity and cannot exceed the speed of light C .

WHEN SPACE EXPANDS, LIGHT STRETCHES

Since the big bang, the physical space of the universe has been expanding. Stars and galaxies maintain their size, but the space *between* them grows.



When galaxies are carried along with expanding space, the wavelength stretches longer and never gets shorter. This is not the same as the Doppler Effect due to galaxy motion through space. Galaxy motion with expanding space or the “Hubble Flow” is called **Cosmological redshift**. According to General Relativity, recession speed of space can exceed light speed.



Measuring redshift: Fifth line from left-very distant galaxy

$$Z = (635 \text{ nm} - 520 \text{ nm}) / 520 \text{ nm}$$

$$Z = 0.221$$

**How do we get recession speed from
redshift?**

How do we get recession speed from redshift?

EQ#1 $V_R = c \left(\frac{da(t)}{dt} \right) \int dz / H(z)$
(Davis)

Flat Homogeneous Isotropic universe
Expansion of space/scale factor
(dark energy) $\Omega_\Lambda = .7$ (matter) $\Omega_m = .3$
large scales > 300 MLy
General Relativity FLRW metric
“no speed limit”

How do we get recession speed from redshift?

EQ#2

$$V_P = c \frac{((1+z)^2 - 1)}{((1+z)^2 + 1)}$$

**Peculiar Velocities within / near galaxy clusters
Due to local gravitational effects**

Governed by Special Relativity $V_P < c$

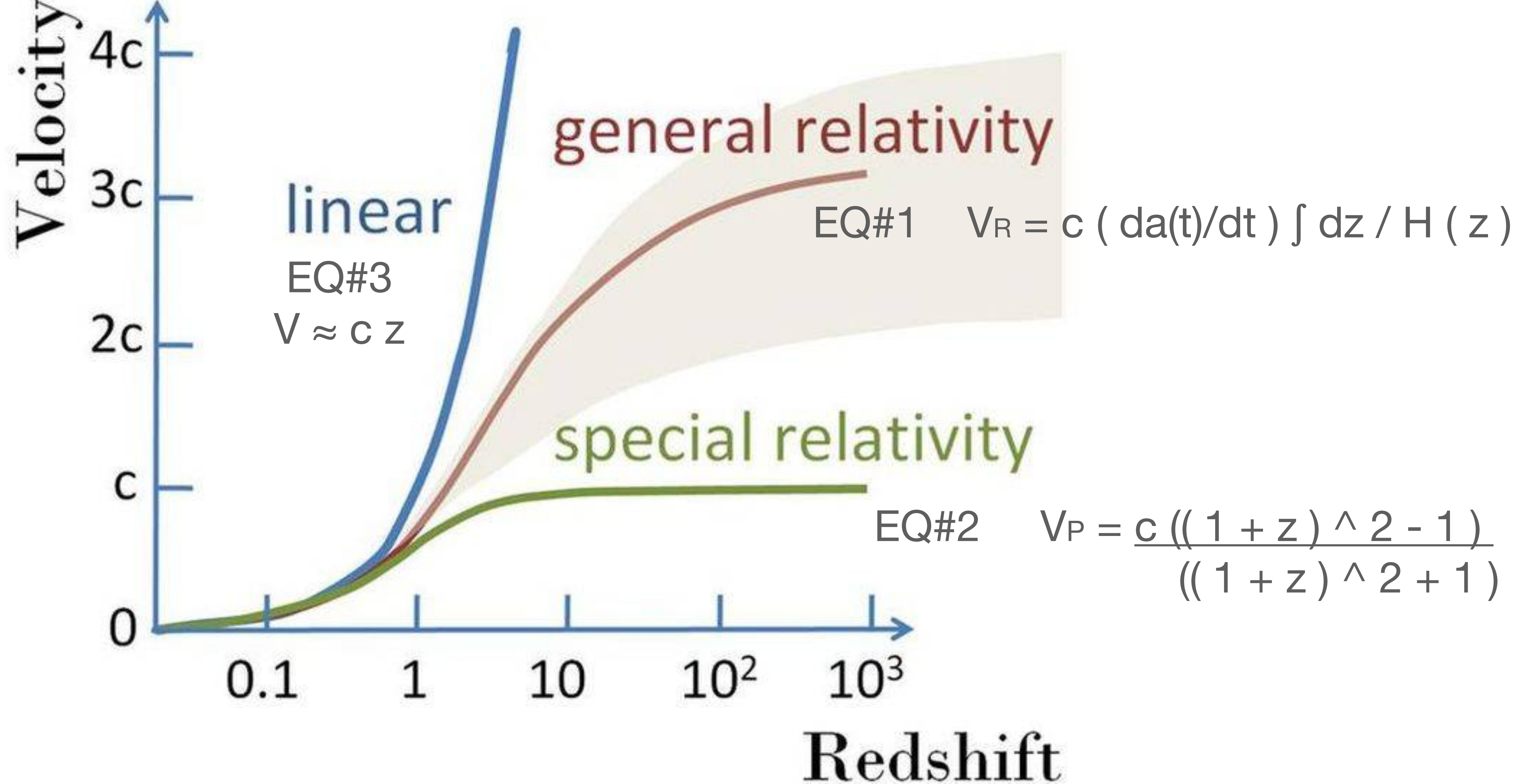
How do we get recession speed from redshift?

EQ#3

$$V \approx cz$$

for $z \leq 0.05$ within 5%

Used by Slipher and Hubble
1920's 1930's



- EQ # 1 expansion of space Λ CDM homogeneous isotropic FLRW metric —
- EQ # 2 peculiar velocities of galaxies through space —
- EQ # 3 approximate radial velocity good for $z \leq 0.1$ —

1929

Edwin Hubble's Great Discovery



Edwin Hubble:
Observational Evidence for an
Expanding Universe 1929

1929

Edwin Hubble's Great Discovery



Slipher + Leavitt = Hubble's Law

Edwin Hubble:
Observational Evidence for an
Expanding Universe 1929

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Edwin Hubble:

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Expanding Universe 1929

Slipher + Leavitt = Hubble's Law



**Used 21 radial
velocities of
spiral nebula
determined by
Vesto Slipher
plus 4 of his
own**

1929

Edwin Hubble's Great Discovery

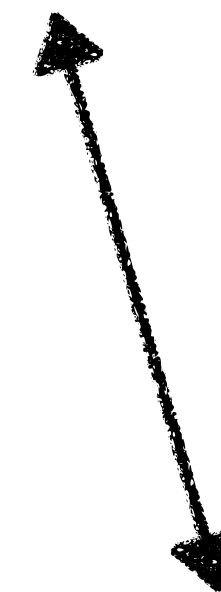


Edwin Hubble:
Observational Evidence for an
Expanding Universe 1929

Slipher + Leavitt = Hubble's Law

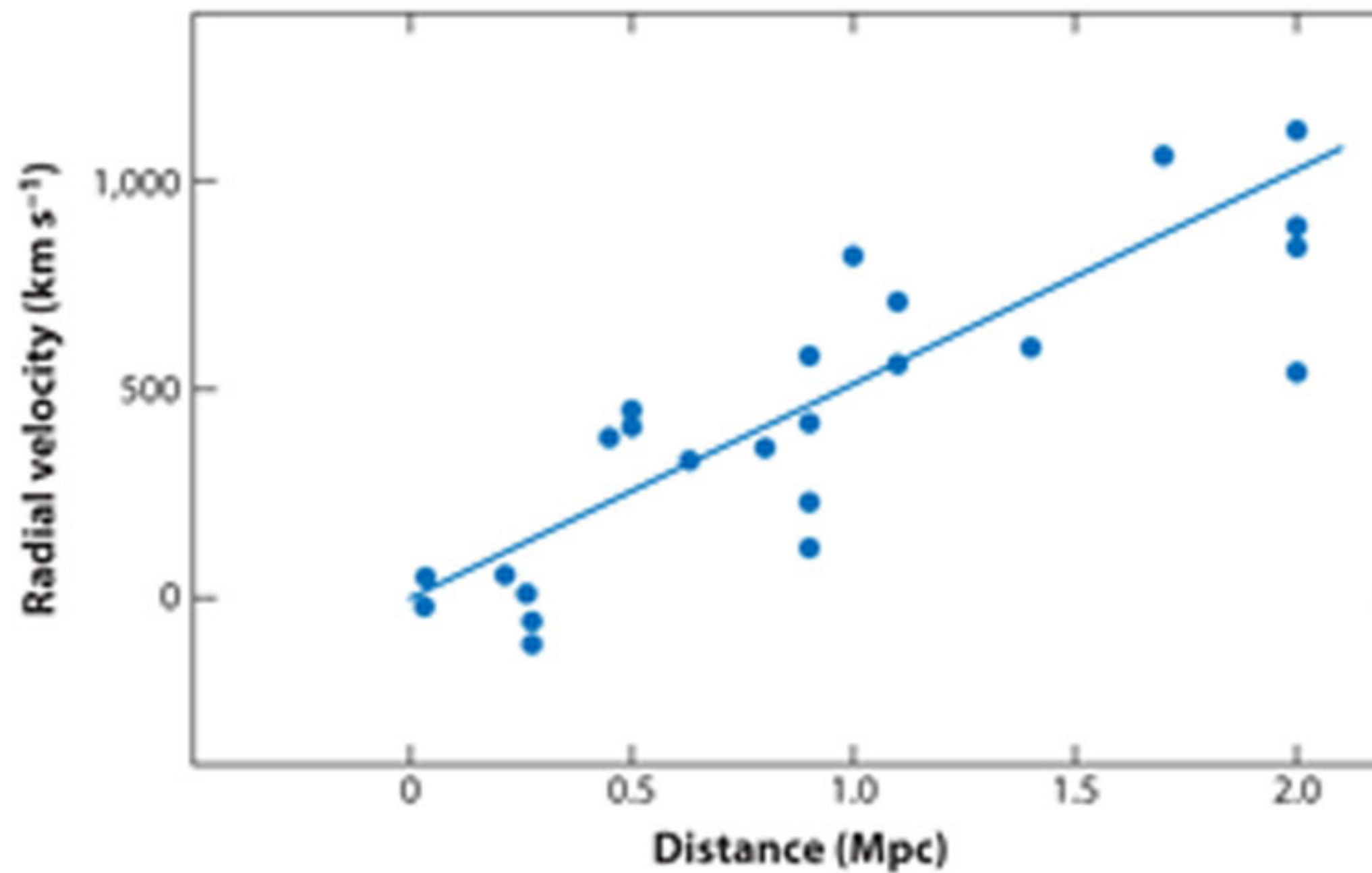


**Used 21 radial
velocities of
spiral nebula
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plus 4 of his
own**



**Used Henrietta Leavitt's
Cepheid "Period - Luminosity"
Law to find distances to the
25 spiral galaxies he knew
radial velocities for**

1929: Edwin Hubble's Great Discovery



Hubble's Great Insight:

Connect radial velocity and distance

$$V \propto D$$

$$V = H \times D$$

H is the slope = 500 km/s/ Mpc

At 1 Mpc, recession is 500 km/s

At 2 Mpc, recession is 1000 km/s

Slipher + Leavitt = Hubble's Law

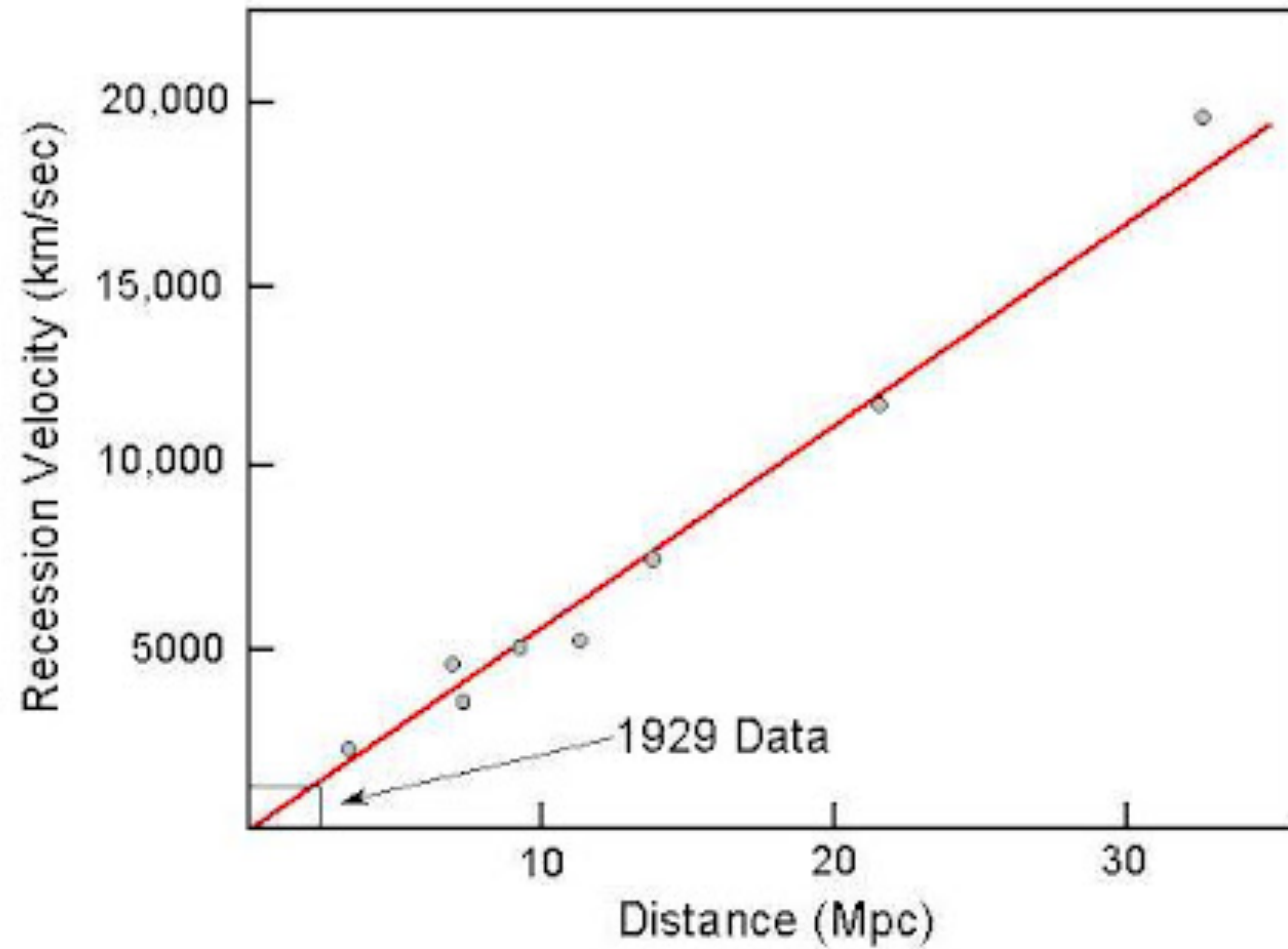
Used 21 radial velocities of spiral nebula determined by Vesto Slipher plus 4 of his own

Used Henrietta Leavitt's Cepheid "Period - Luminosity" Law to find distances to the 25 spiral galaxies he knew radial velocities for

$$V \propto D$$

$$V = H \times D$$

Hubble & Humason (1931)



Hubble's Law holds for much larger distance values in 1931

Hubble's Great Insight:

Connect radial velocity and distance

$$V \propto D$$

$$V = H \times D$$

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Hubble's Great Insight:

Connect radial velocity and distance

$$V \propto D$$

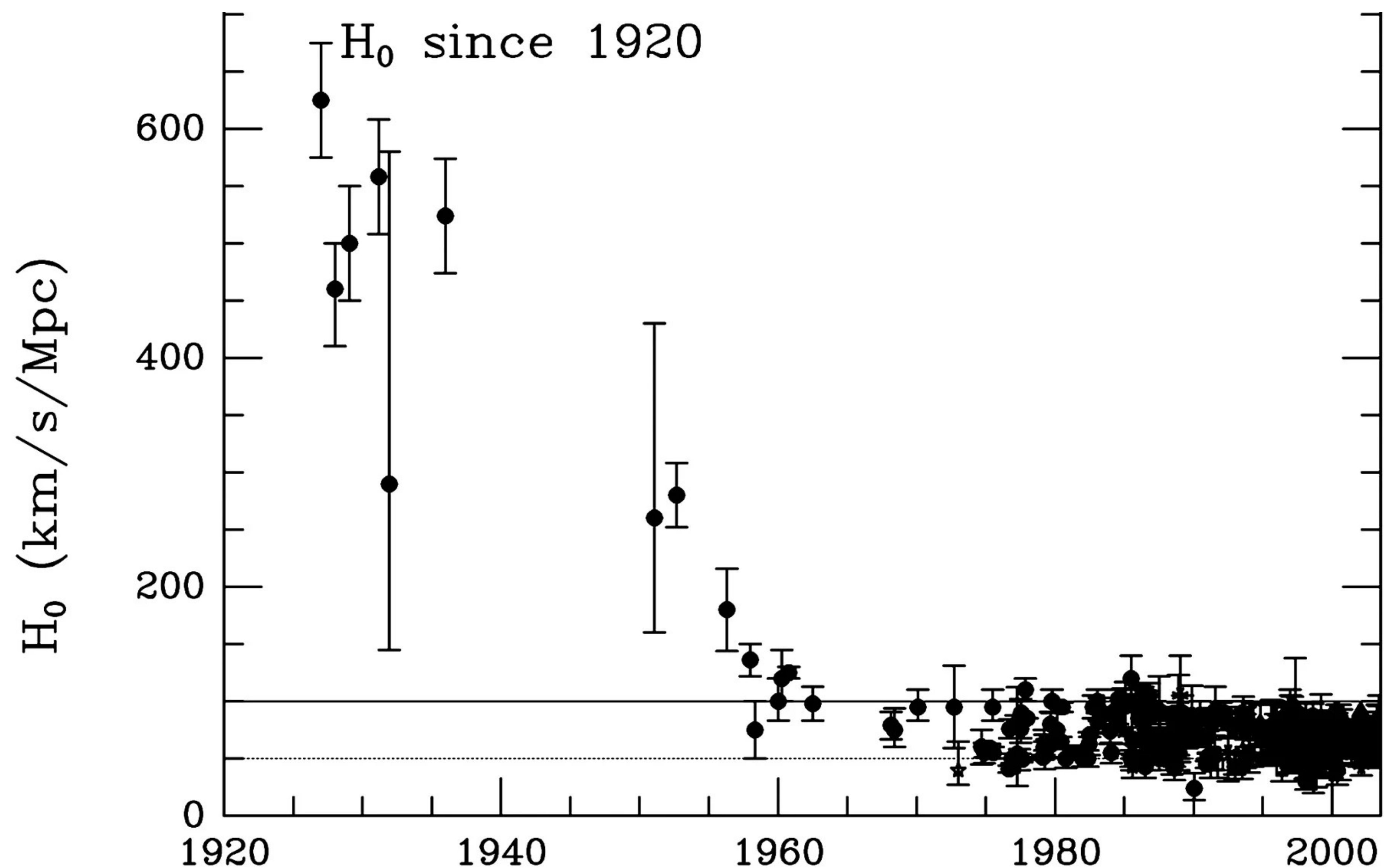
$$V = H \times D$$

H is the slope = 500 km/s/ Mpc

At 1 Mpc, recession is 500 km/s

At 2 Mpc, recession is 1000 km/s

Hubble did not know there were two types of Cepheids. As a result his value for the “Hubble constant now” H_0 was about seven times too big. However, his work was strong evidence for a universe that was **not static** as once believed, but **expanding** in scale!



With better technology, space telescopes, and better models,
measurement of “Hubble’s constant **H₀** now” is now about
70 km/s/Mpc

What does 70 km/s/Mpc mean?

What does 70 km/s/Mpc mean?

#1) A galaxy 100 Mpc from us now is moving at

_____ km/s

What does 70 km/s/Mpc mean?

#1) A galaxy 100 Mpc from us now is moving at

7000 km/s

What does 70 km/s/Mpc mean?

#2) A galaxy 200 Mpc from us now is moving at

_____ km/s

What does 70 km/s/Mpc mean?

#2) A galaxy 200 Mpc from us now is moving at

14000 km/s

What does 70 km/s/Mpc mean?

#2) A galaxy 5000 Mpc or 5 Gps from us now is moving at

_____km/s

What does 70 km/s/Mpc mean?

#2) A galaxy 5000 Mpc or 5 Gpc from us now is moving at

350 000 km/s

What does 70 km/s/Mpc mean?

**#2) A galaxy 5000 Mpc or 5 Gpc from us now is moving at
350 000 km/s**

Note:

When we use H_0 this way, we think of the galaxy moving away from us because it is being carried away with expanding space!

According to General Relativity, **space has no speed limit when it expands** and it can expand faster than “c” the speed of light

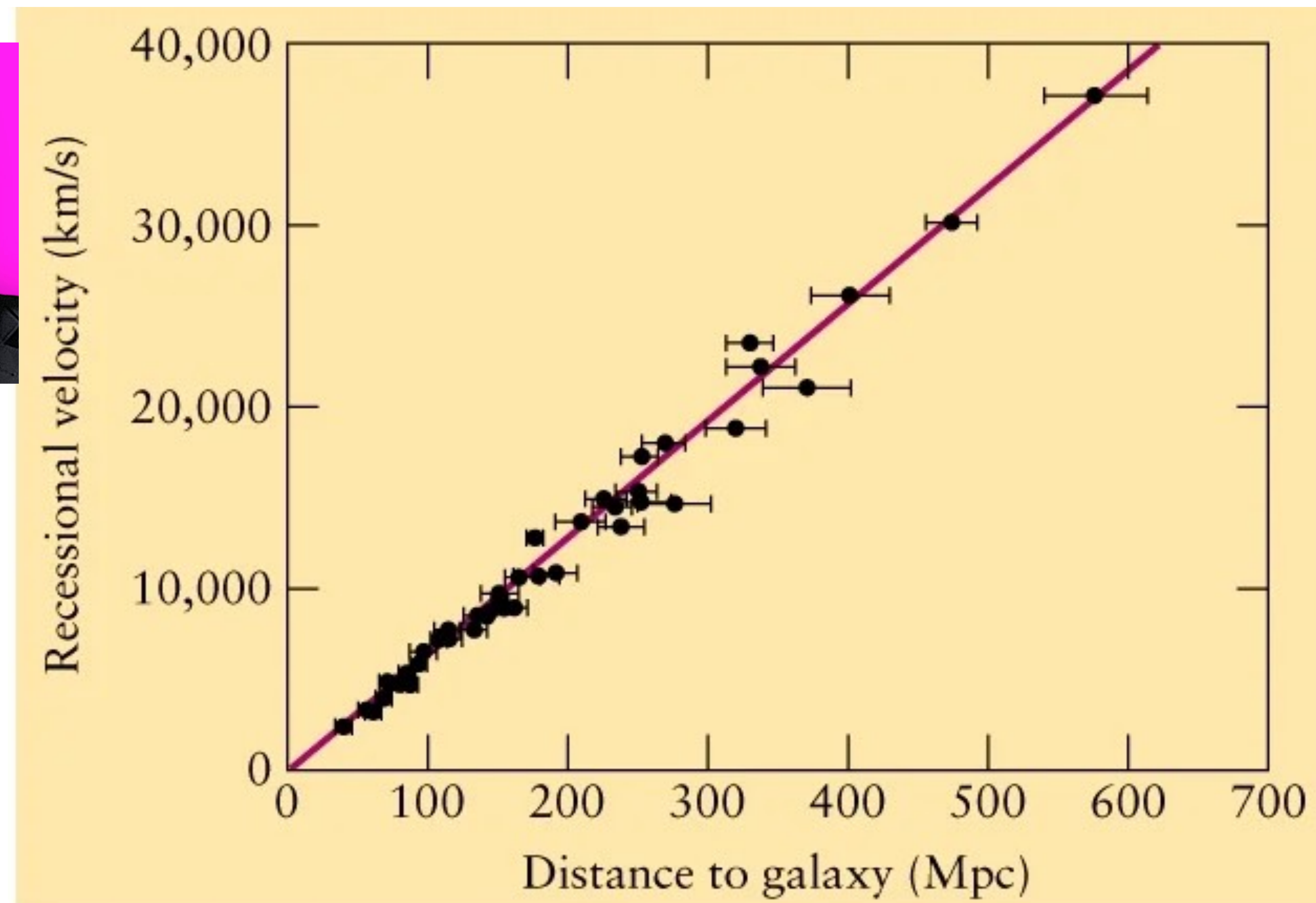
Velocities of galaxies through space are called **peculiar velocities** due to local gravitational effects. Peculiar velocities are subject to Special Relativity and **cannot exceed light speed “c”**

Hubble’s law becomes more valid as proper distance (the distance now) between our Milky Way and other galaxies become large, say, greater than 100 Mpc.

At distances less than 5 Mpc, peculiar velocity is comparable to the speed of expanding space and Hubble’s Law is less valid. Our Andromeda galaxy is 0.77 Mpc away and is actually moving toward the centre of our Milky Way



Let's find H_0 ?



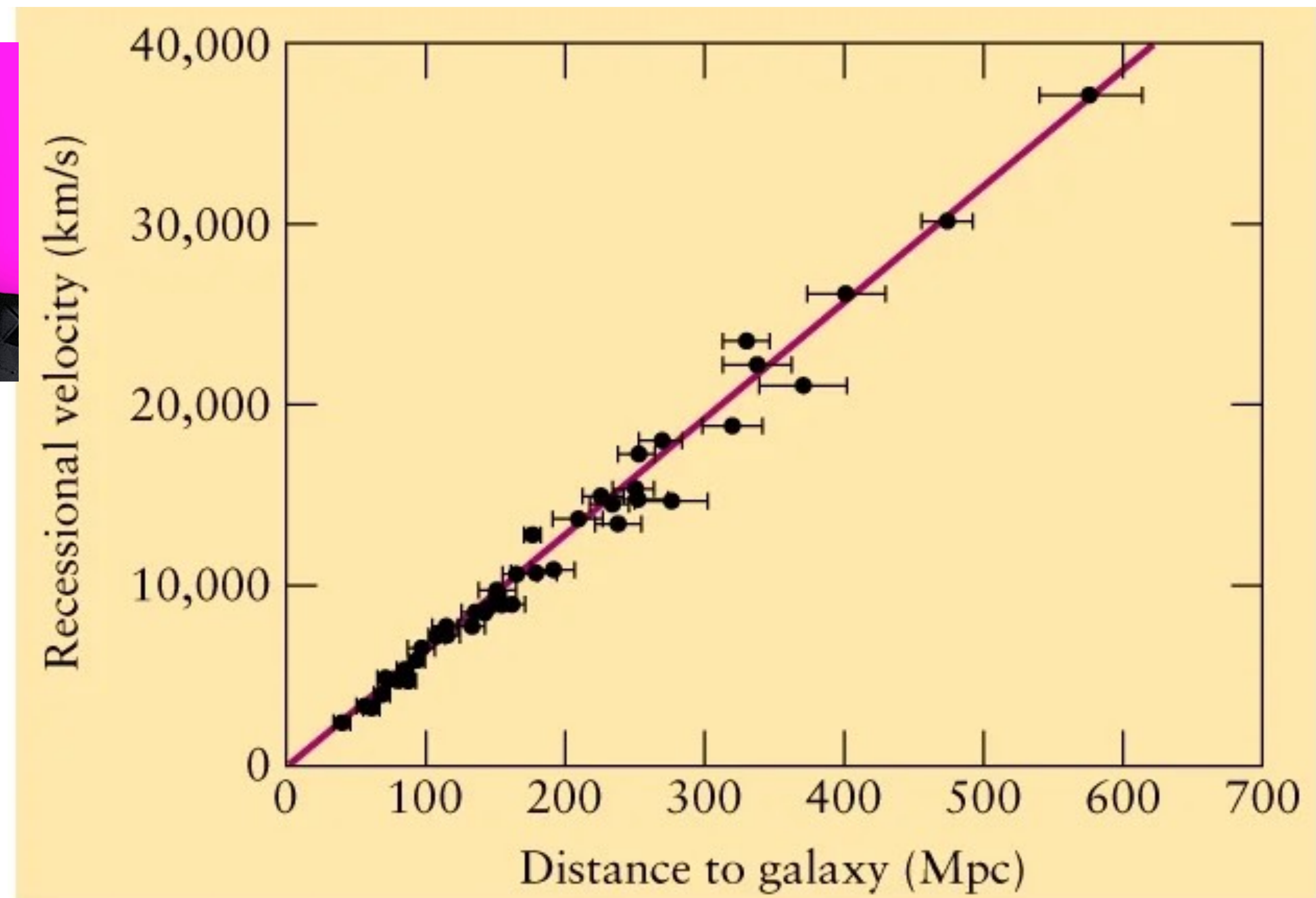
Adam Riess

Hubble constant graph

John Hopkins University SHoES project 2021



Let's find H_0 ?
 H_0 = slope



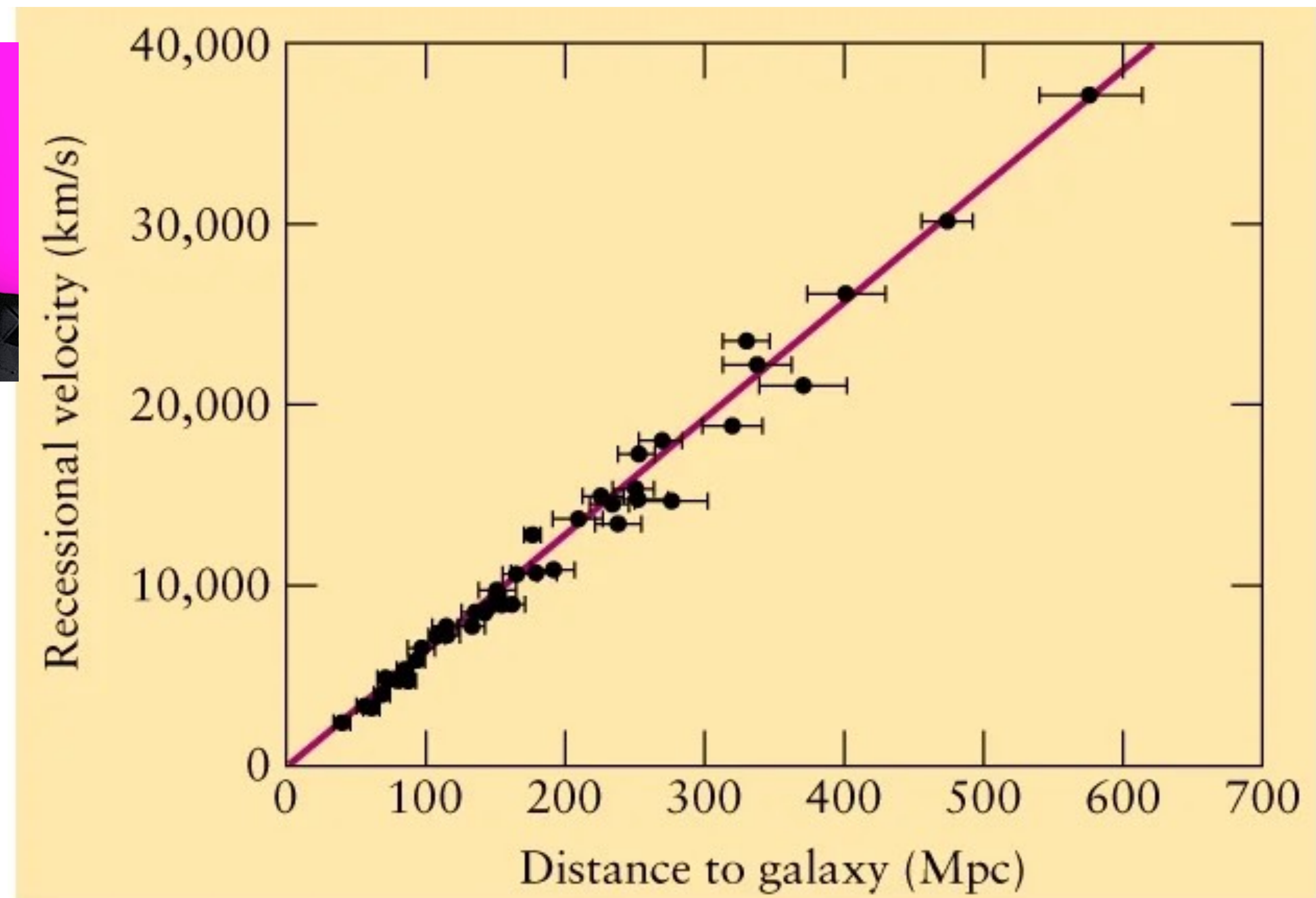
Adam Riess

Hubble constant graph

John Hopkins University SHoES project 2021



Let's find H_0 ?
 $H_0 = \text{slope}$
 $= \frac{40\,000 \text{ km/s}}{620 \text{ Mpc}}$



Adam Riess

Hubble constant graph

John Hopkins University SHoES project 2021

COSMIC CRISIS

ADAM RIESS



Let's find H_0 ?

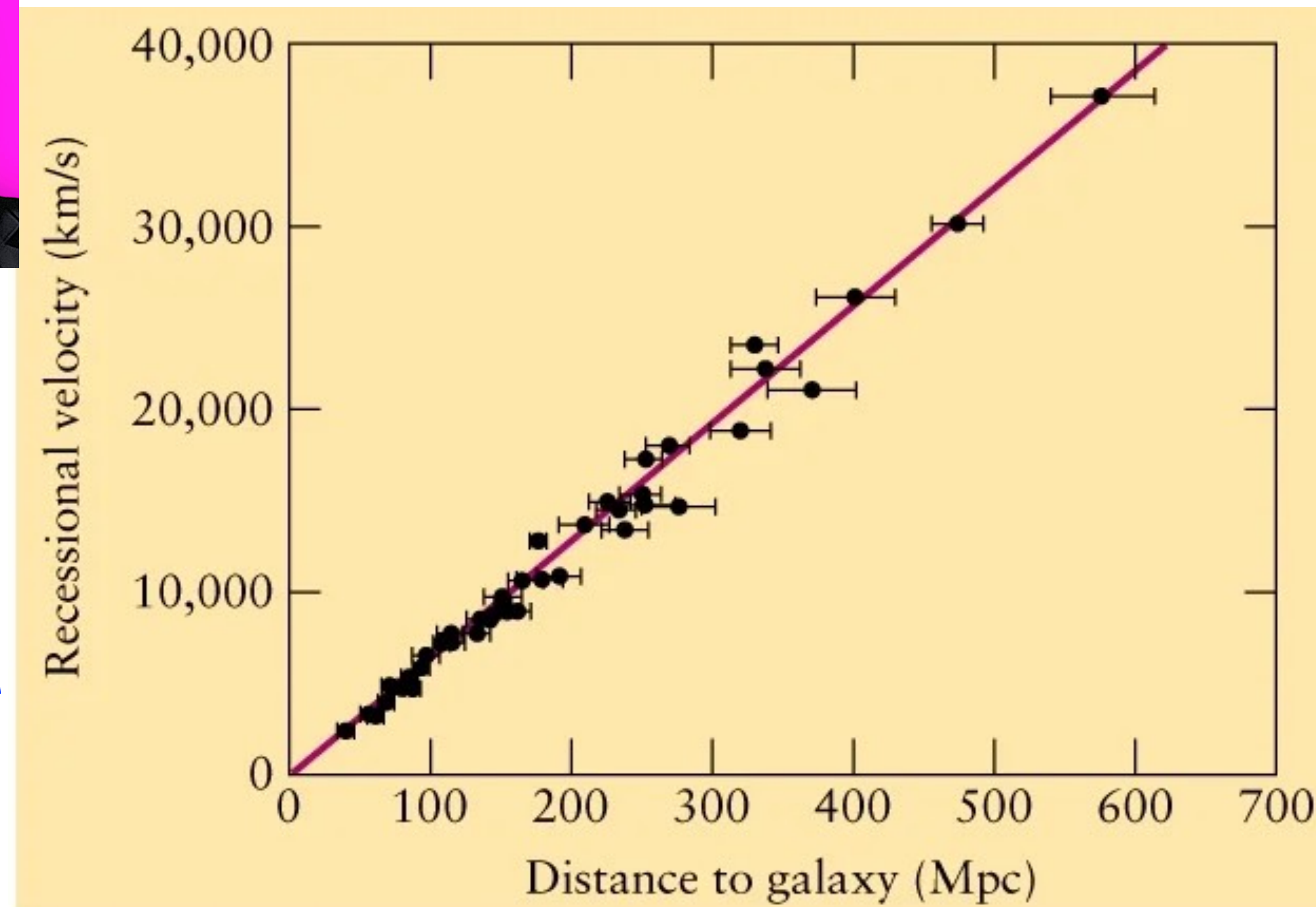
H_0 = slope
= 40 000 km/s

620 Mpc

But **620 Mpc** is a
luminosity distance
assuming a **static**
universe

We want a proper
distance!

(Expanding
universe)



Adam Riess

Hubble constant graph

John Hopkins University SHoES project 2021

**As light waves lengthen they lose
energy ...**

**Light arriving to us appears less bright
and things look farther away than they
actually are ...**

**So astrophysicists use a formula to
calculate actual proper distance from
luminosity distance**

$$\begin{aligned} &\text{Proper distance} \\ &= \\ &\text{luminosity distance} / (1 + \text{redshift or } z) \end{aligned}$$

Proper distance
= luminosity distance / (1 + redshift or z)

= 620 Mpc (1 + 0.132)
assuming 70% dark energy and 30% matter

**Proper distance
= luminosity distance / (1 + redshift or z)**

**= 620 Mpc (1 + 0.132)
assuming 70% dark energy and 30% matter**

= 547.7 Mpc

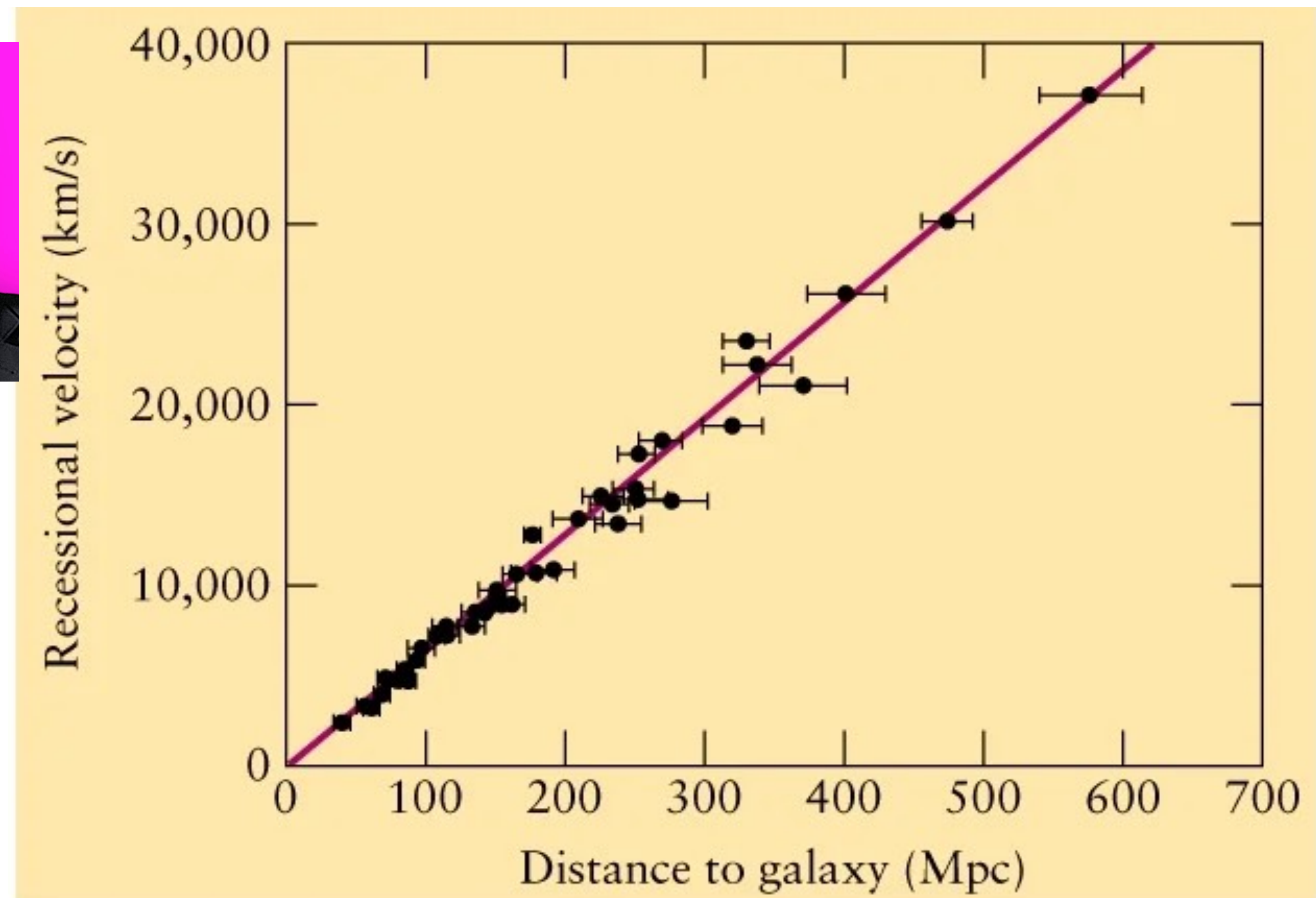


Let's find H_0 ?

$H_0 = \text{slope}$
 $= \underline{40\,000 \text{ km/s}}$

620 Mpc
 $= \underline{40\,000 \text{ km/s}}$

547.7 Mpc
Corrected for
expansion



Adam Riess

Hubble constant graph

John Hopkins University SHoES project 2021



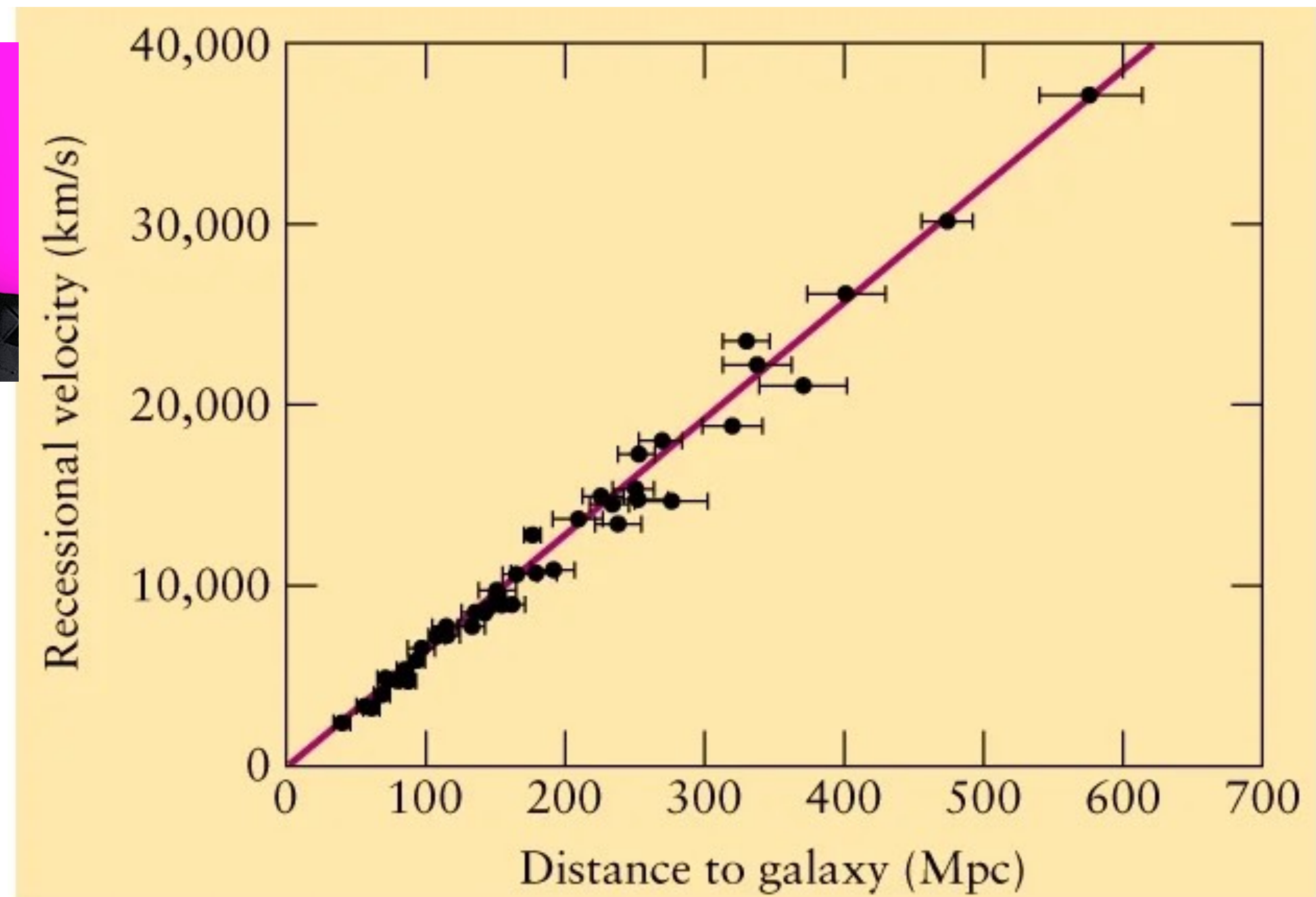
Let's find H_0 ?

$H_0 = \text{slope}$
 $= \frac{40\,000 \text{ km/s}}{620 \text{ Mpc}}$

$= \frac{40\,000 \text{ km/s}}{547.7 \text{ Mpc}}$

Corrected for expansion

$= 73.0 \text{ km/s/Mpc}$



Adam Riess

Hubble constant graph

John Hopkins University SHoES project 2021



$H_0 = 73.0 \text{ km/s/Mpc}$
 ± 1.0

Used Cepheids and
type 1a supernova
as standard candles

John Hopkins

SHoES

2021



$H_0 = 73.0 \text{ km/s/Mpc}$
 ± 1.0

Used Cepheids and
type 1a supernova
as standard candles

John Hopkins

SHoES

2021



Wendy Freedman
**Carnegie Chicago Hubble
Program 2025**

$H_0 = 70.4 \text{ km/s/Mpc}$
 ± 1.9

Used Tip of Red Giant
Branch (see Hertzsprung-
Russel diagrams for more
info)

and type 1a supernovas as
standard candles

Measuring H_0 using the CMB

Measuring H_0 using the CMB

What is the CMB?

Measuring H_0 using the CMB

What is the CMB?

Oldest light in the universe

Light has been **stretched** for 13.8 GLy

Was visible/infrared but is **now microwave**

Detected **everywhere** in the sky

Very **Uniform** with 1 in 10^5 density variation

(COLD and HOT spots)

Cosmic **M**icrowave **B**ackground

**When and how did the CMB
form?**

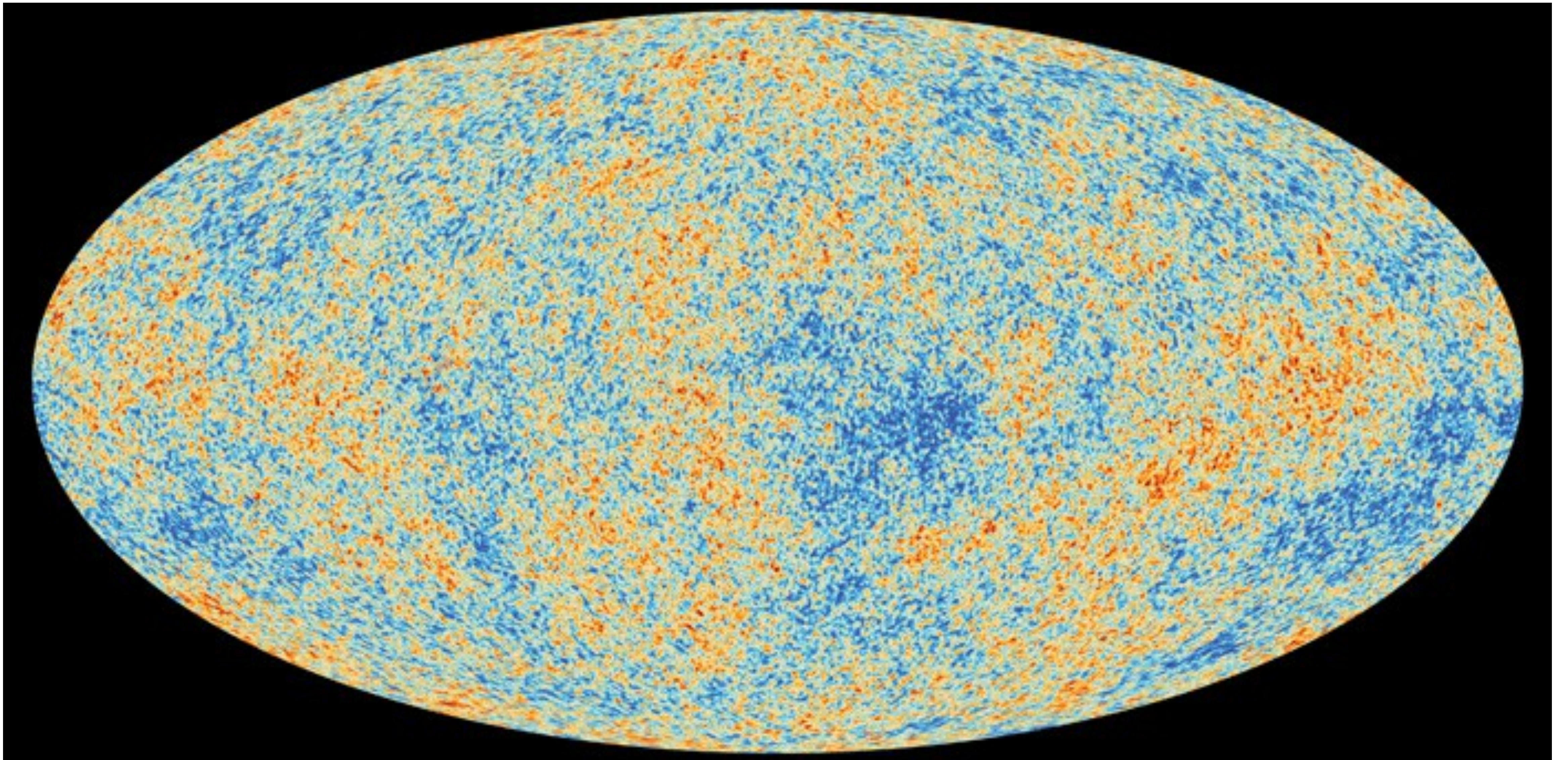
When and how did the CMB form?

Light released 380 000 years after the big bang

Plasma of protons and electrons expanded and cooled enough to allow them to come together to form neutral atoms 3000 K

Era of Recombination

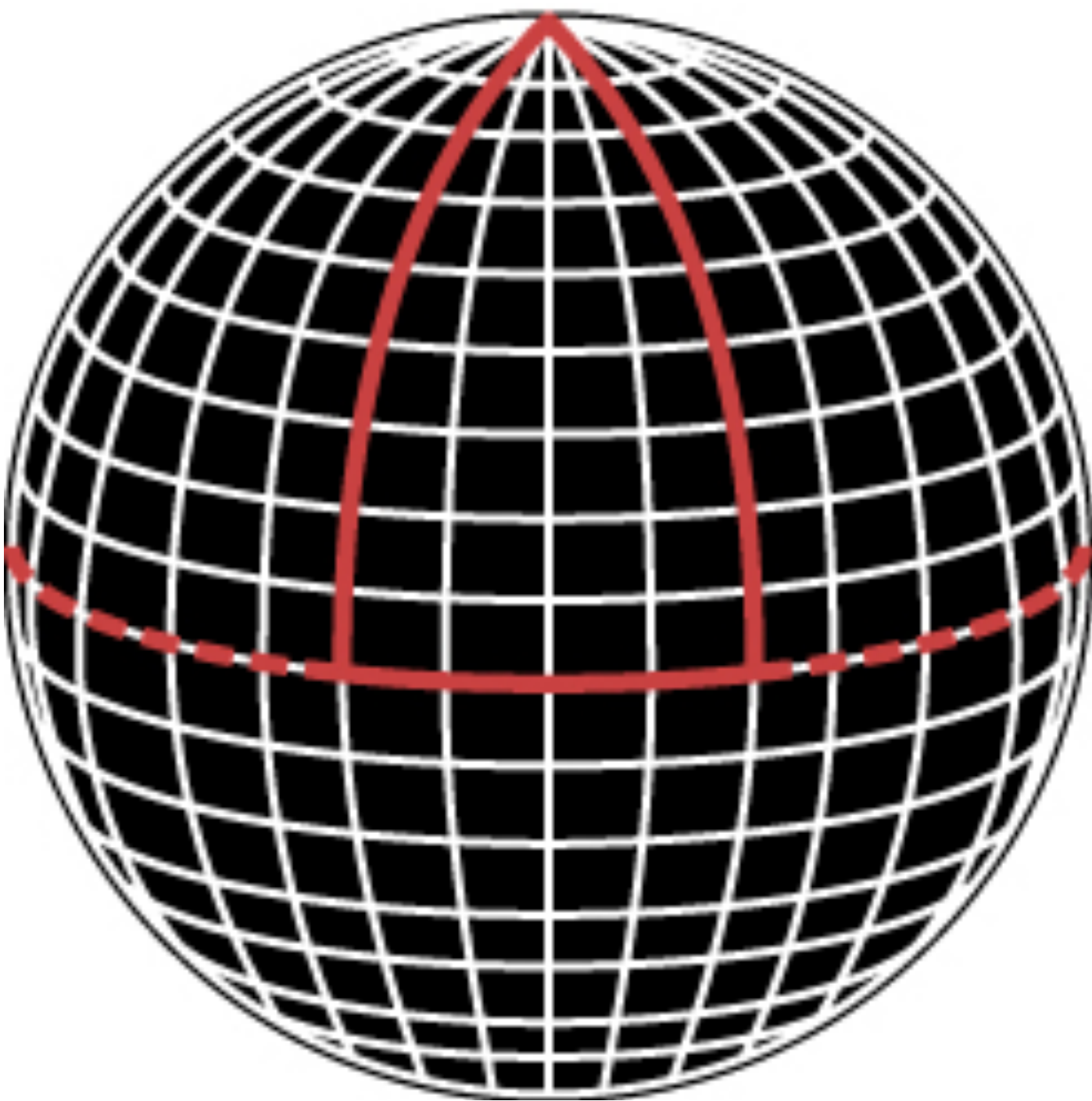
Made space transparent to light



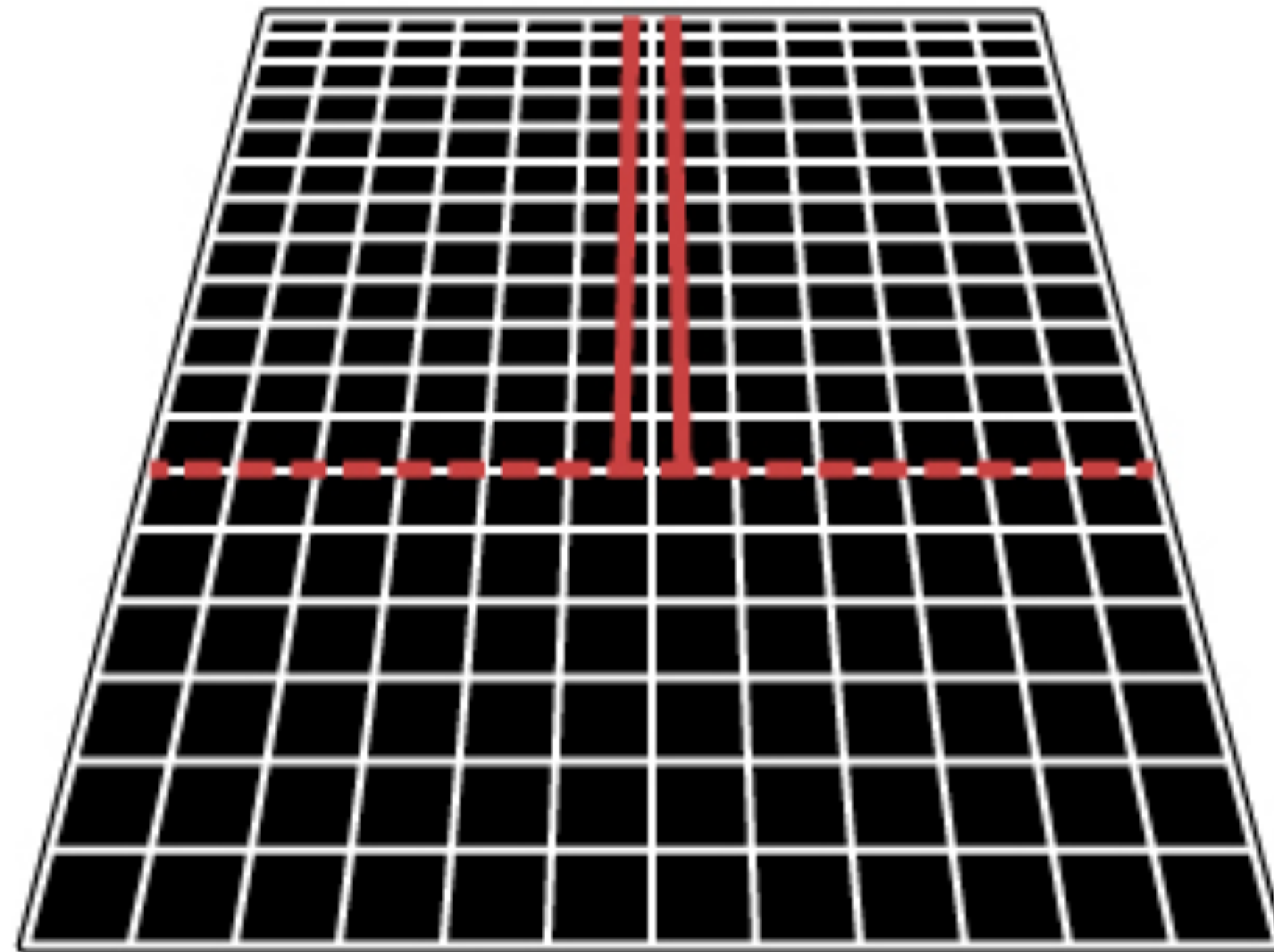
Planck Satellite Microwave Map: [ESA](#)

By studying the size and relative frequency of the more common spots H_0 and curvature can be determined

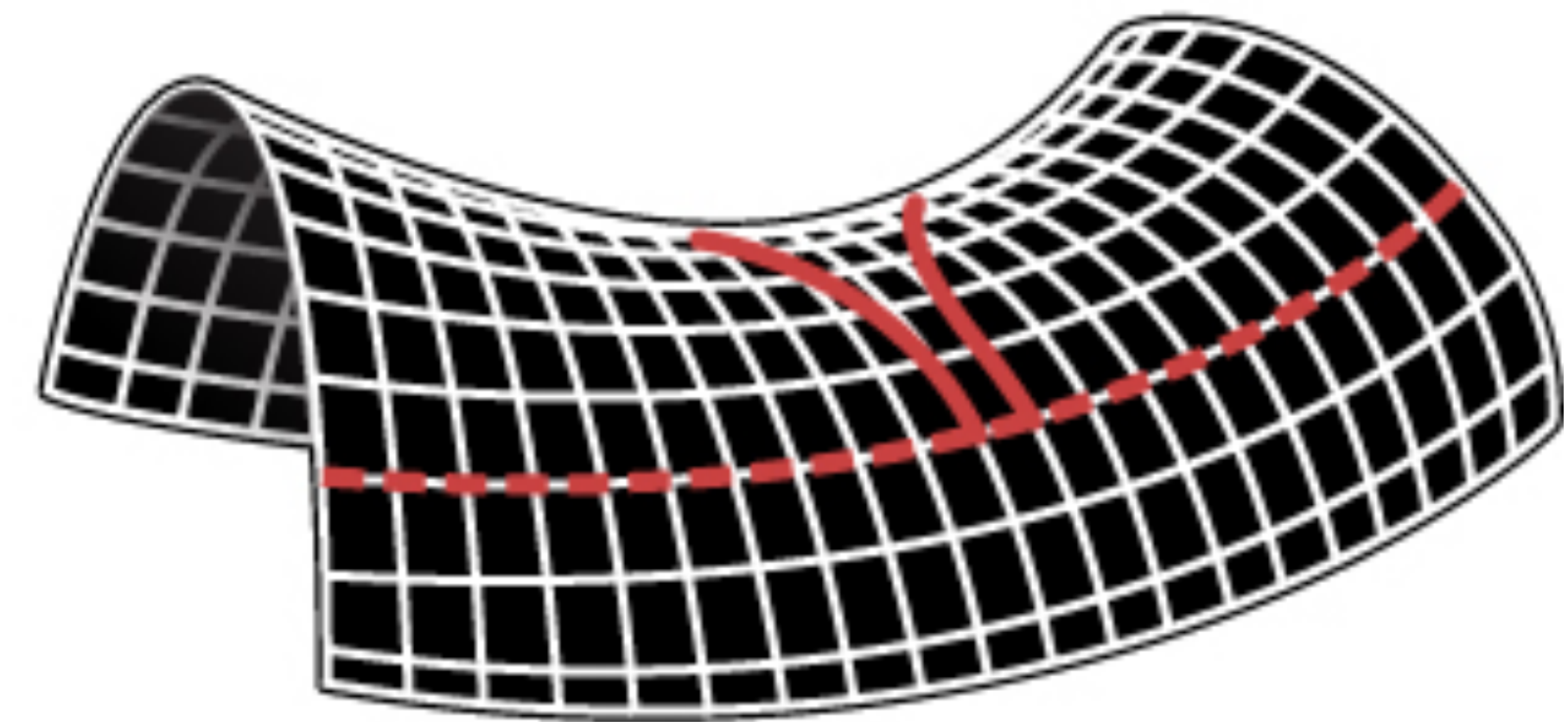
Spherical space



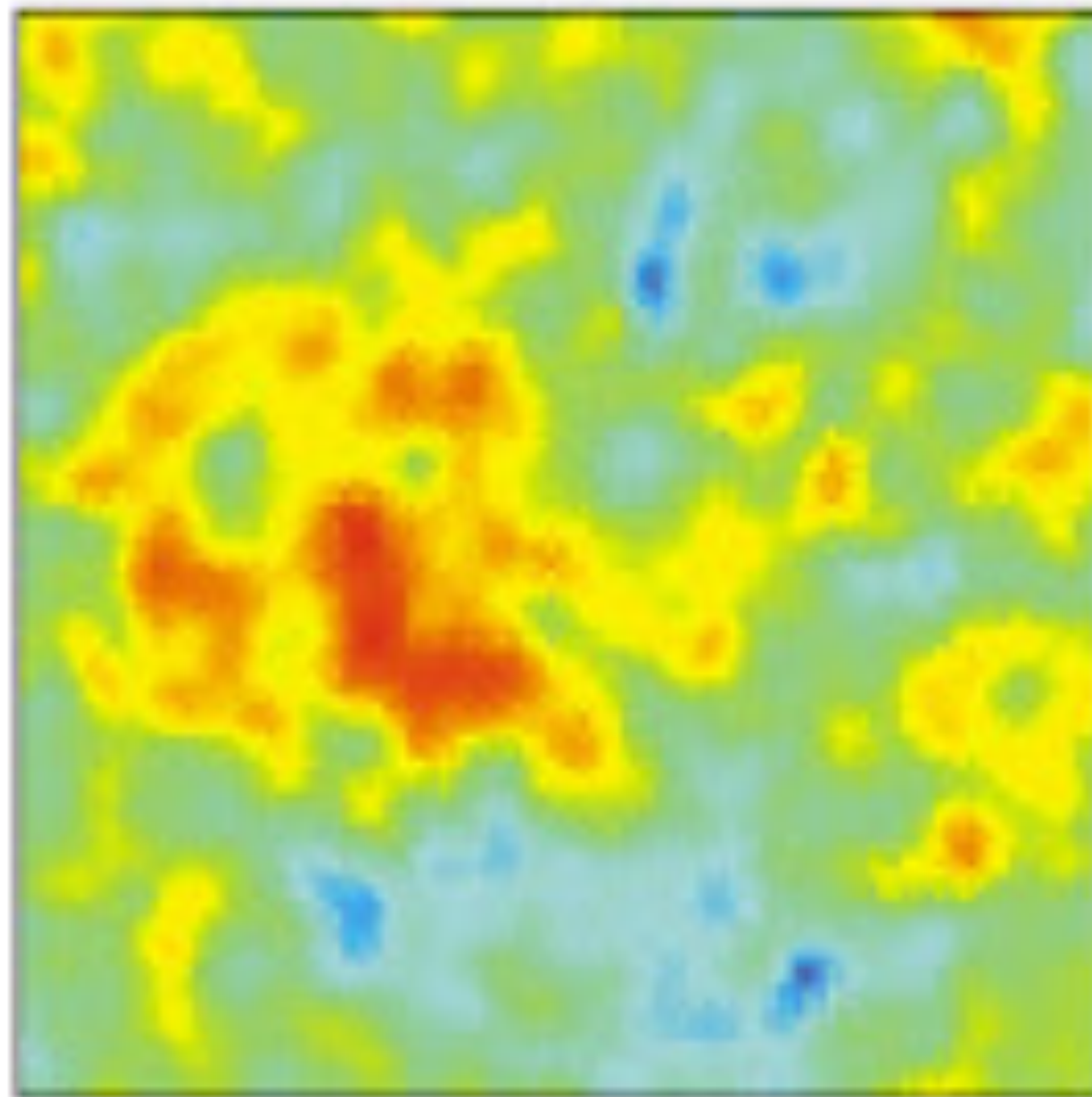
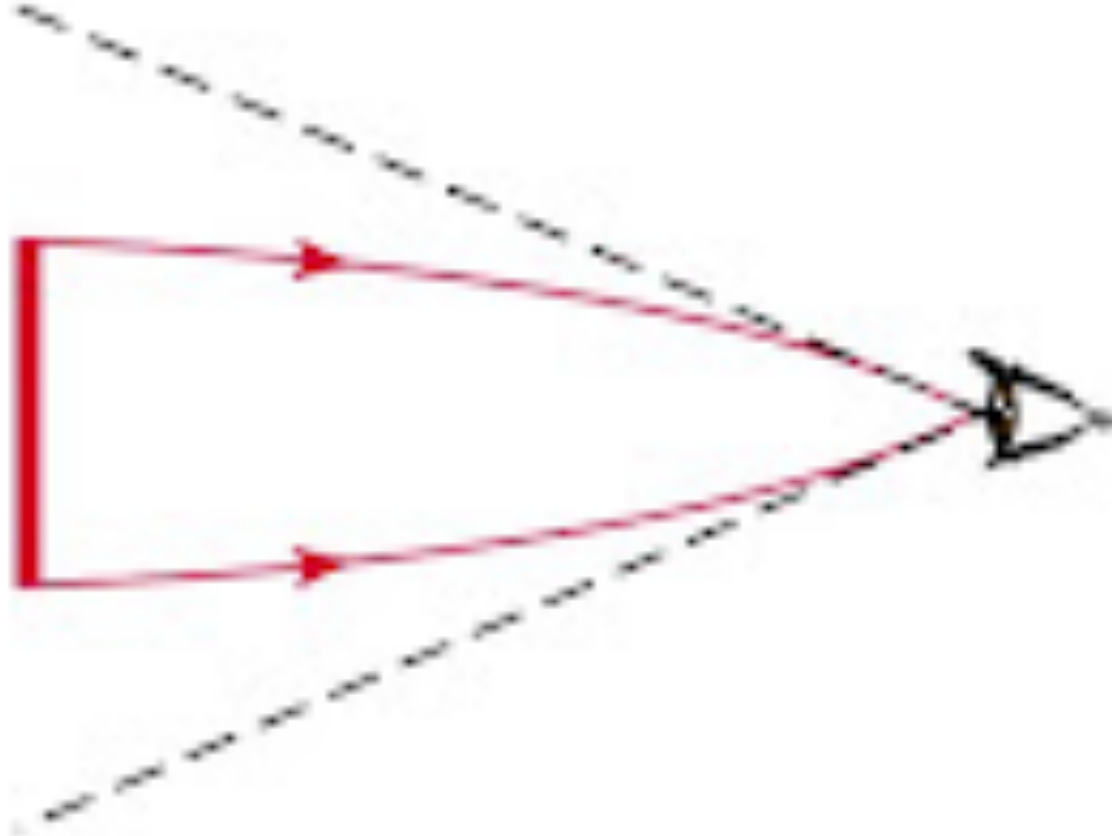
Flat space



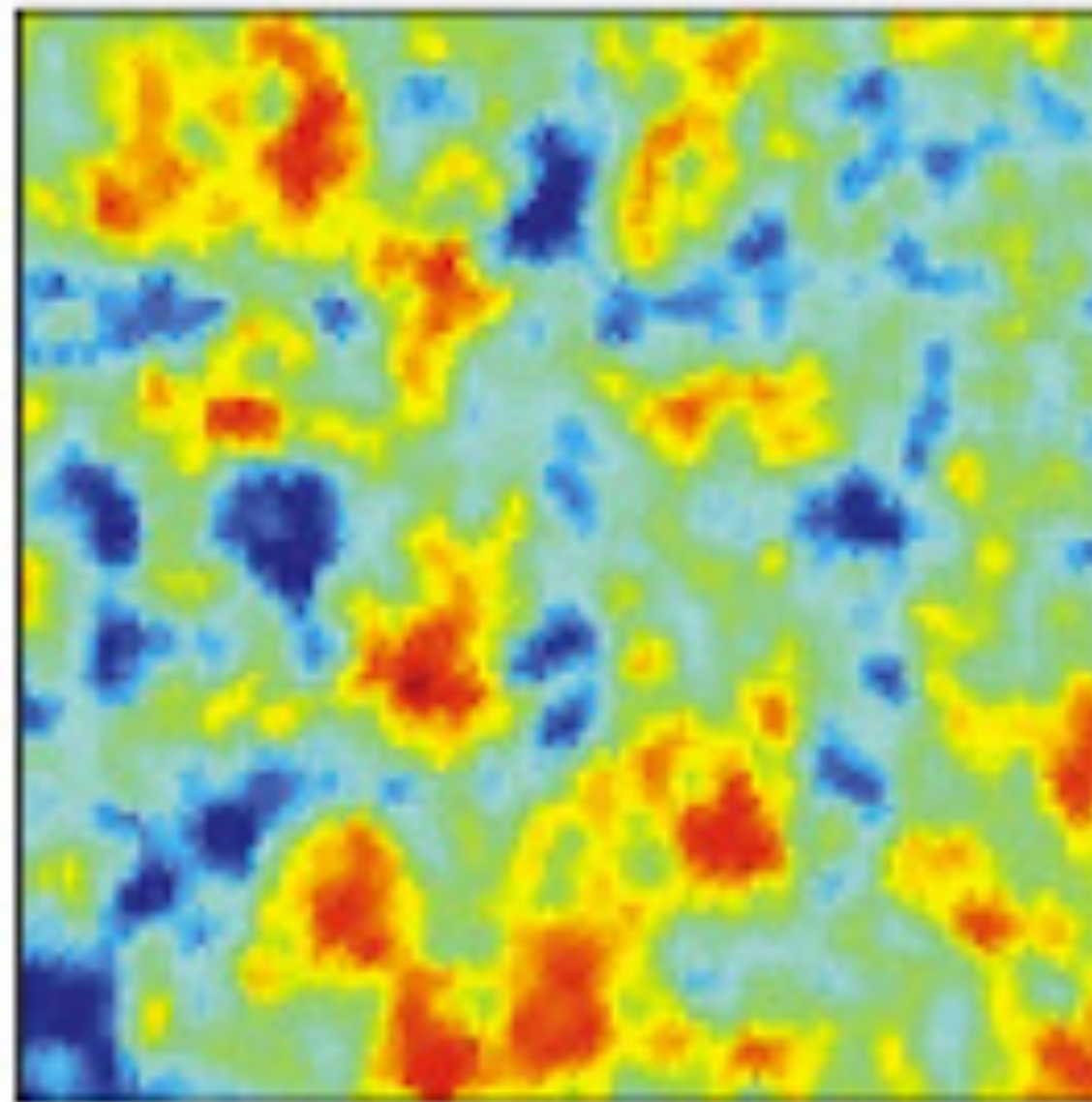
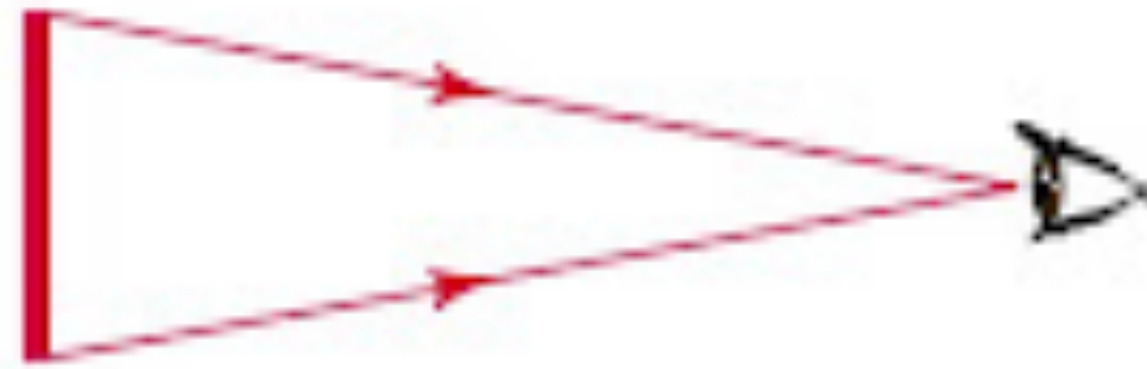
Hyperbolic space



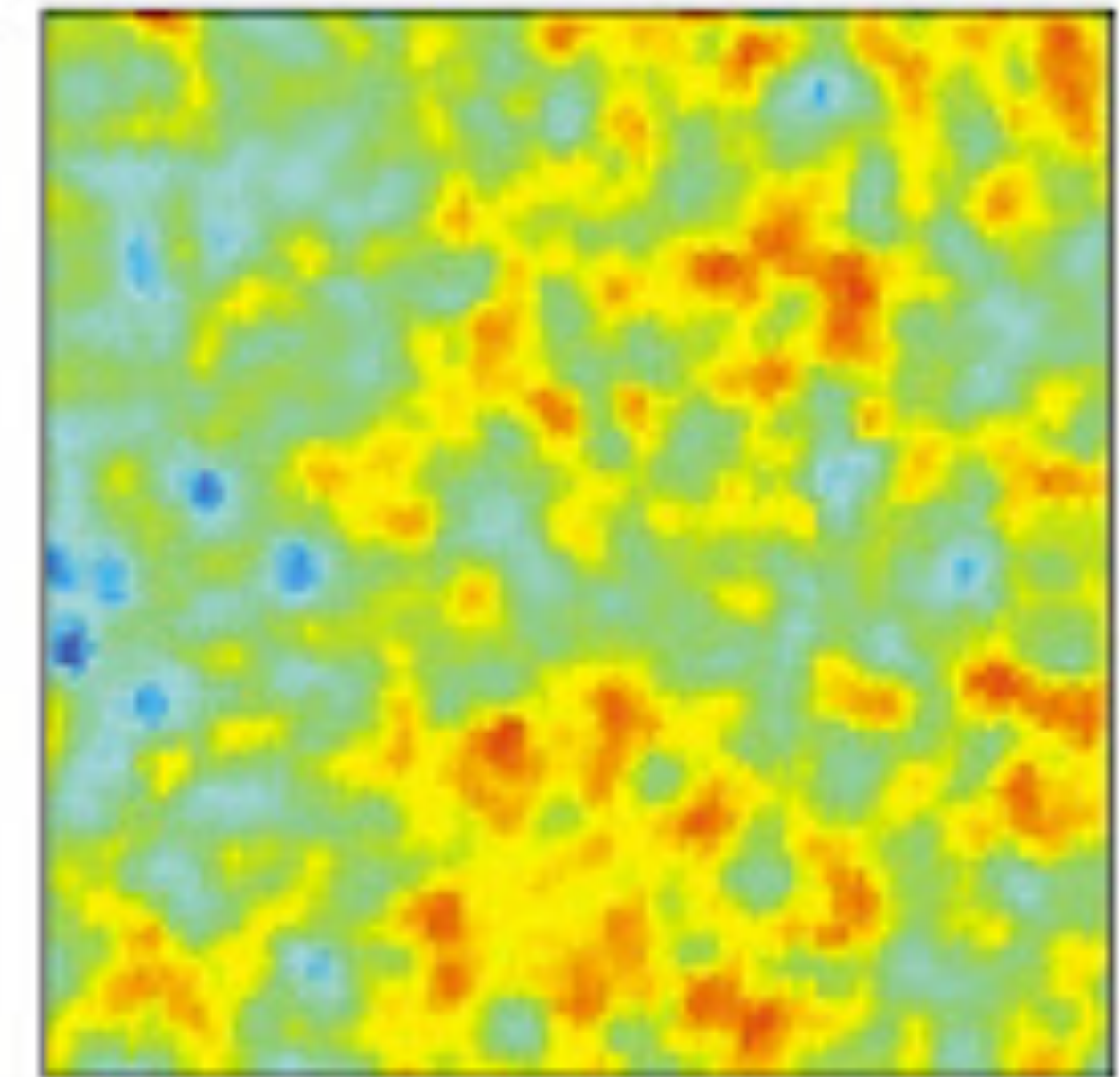
Measuring the Cold and dark spots of the CMB helps determine the geometry of the universe and Hubble's constant then and now



a If universe is closed,
“hot spots” appear
larger than actual size



b If universe is flat,
“hot spots” appear
actual size



c If universe is open,
“hot spots” appear
smaller than actual size

“Hot spots” are areas of more mass and greater density that later form galaxies and galaxy clusters
The Planck satellite found that most spots appeared actual size of about 1 degree,
indicating a flat universe with no curvature

**What were the results of the
Planck Satellite CMB mapping?**

What were the results of the Planck Satellite CMB mapping?

Strong evidence of a **flat universe** by looking at the size of density fluctuations, as they were the same size as predicted by sound standing waves in a plasma

Observations of the relative frequency and size of hot spots in the CMB matched the **Λ -CDM** model of the universe

$$\Omega \text{ (critical density)} = \Omega_m + \Omega_\Lambda = 1 \text{ (flat)}$$

Approximately $\Omega_m = 0.3$ (normal and dark matter)

Approximately $\Omega_\Lambda = 0.7$ (dark energy)

Using the data from the Planck CMB mapping and how it matched the above Λ -CDM model,

the value of the Hubble-Lemaitre parameter was calculated 380 000 years After the Big Bang and extrapolated to the present as

$$H_0 = 67.4 \text{ km/s/Mpc. } \pm 0.5$$

Unresolved Definite Disagreement for H_0 : “Hubble Trouble” or “Tension”



$$H_0 = 73.0 \text{ km/s/Mpc} \pm 1.0$$

Used Cepheids and
type 1a supernova
as standard candles

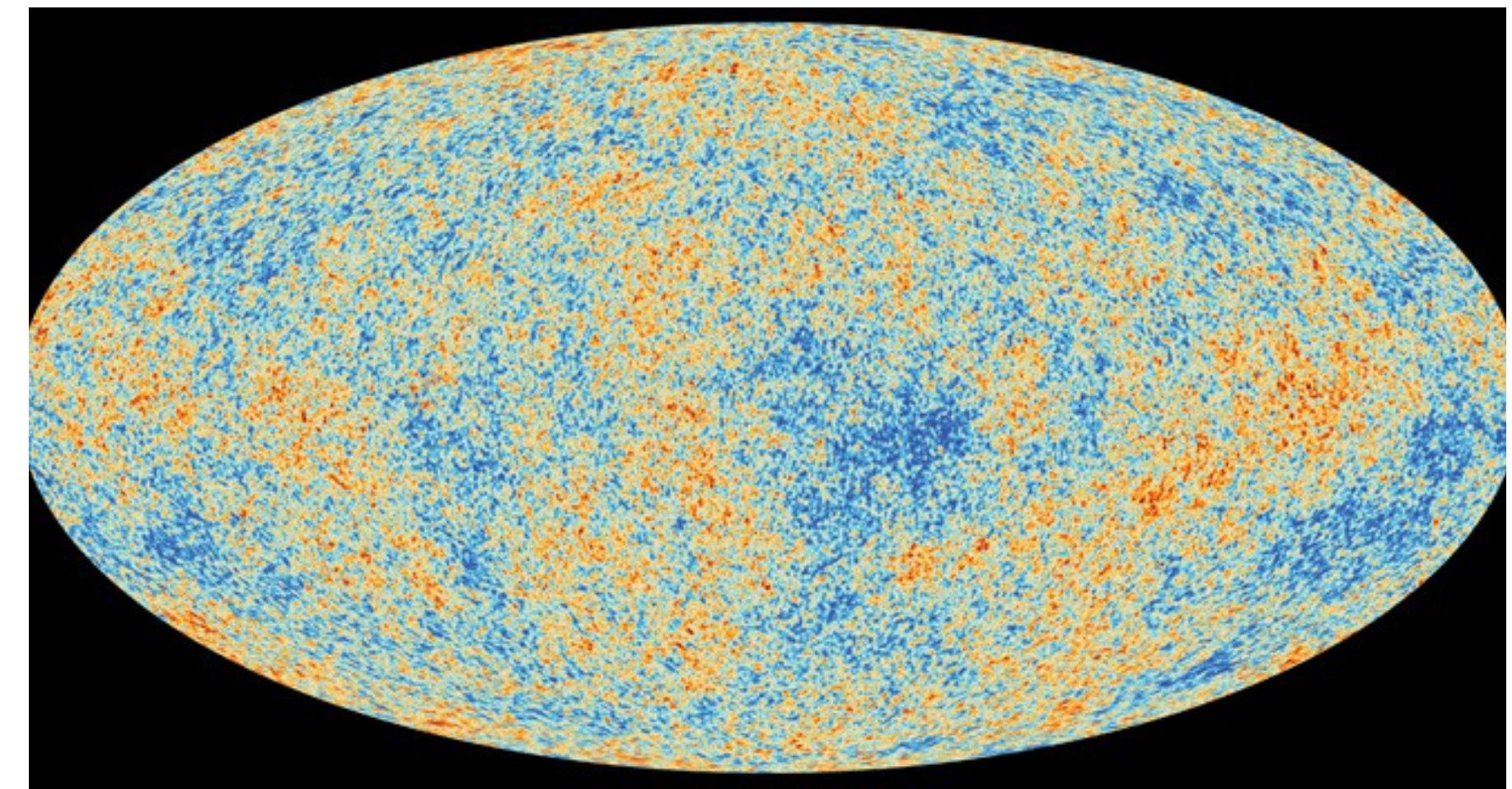
John Hopkins
SHoES
2021



Wendy Freedman
Carnegie Chicago Hubble
Program 2025

$$H_0 = 70.4 \text{ km/s/Mpc} \pm 1.9$$

Used Tip of Red Giant
Branch (see Hertzsprung-
Russel diagrams for more
info)
and type 1a supernovas as
standard candles



Planck Collaboration 2018

Used the ESA Planck satellite to
measure relative frequency and size
of hot spots in the CMB to find the
best fit match with Λ -CDM model

$$H_0 = 67.4 \text{ km/s/Mpc} \pm 0.5$$

**How are “Hubble’s Law” and the
“expansion rate” related?**

How are “Hubble’s Law” and the “expansion rate” related?

$$H_0 = V_R / D_P$$

Hubble’s law from observations



$$H(t) = \frac{da(t)}{dt} / a(t)$$

Defining equation ?

How are “Hubble’s Law” and the “expansion rate” related?

$$H(t) = V_{R(t)} / D_P(t)$$

Hubble’s law from observations



$$H(t) = \frac{da(t)}{dt} / a(t)$$

Defining equation ?

$$H(t) = \frac{da(t) / dt}{a(t)}$$

Note: $H(t)$ is actually a
fractional expansion
rate

Hubble's
Constant

$$H(t) = \frac{da(t) / dt}{a(t)}$$

**Hubble's
Constant**

$$H(t) = \frac{da(t) / dt}{a(t)}$$

**Hubble-Lemaitre
Parameter**

Hubble's
Constant

$$H(t) = \frac{da(t)}{dt}$$

Hubble-Lemaitre
Parameter

(t) means “depends
on time”



The diagram consists of three magenta arrows. One arrow points from the text '(t) means “depends on time”' to the 'H(t)' term in the equation. A second arrow points from the same text to the 'a(t)' term in the denominator of the fraction. A third arrow points from the 'a(t)' term in the denominator to the 'da(t)' term in the numerator of the fraction.

Hubble's
Constant

$$H(t) = \frac{da(t)}{dt}$$

Hubble-Lemaitre
Parameter

(t) means “depends
on time”

Scale factor: compares the
relative size of the
observable universe now, in
the past, and in the future

**Hubble's
Constant**

Expansion rate: how quickly
the scale factor or “space
itself” is expanding

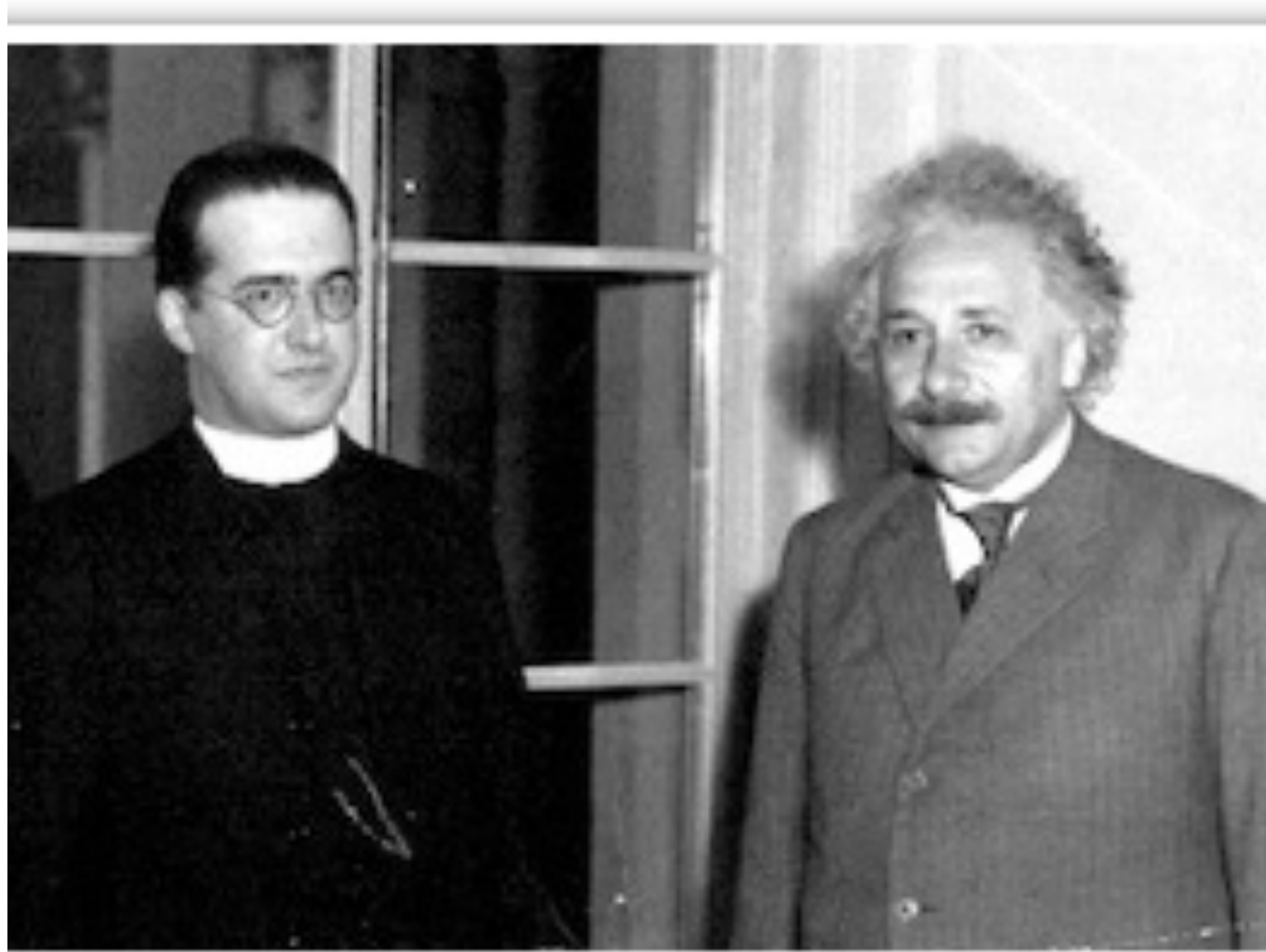
$$H(t) = \frac{da(t)}{dt}$$

**Hubble-Lemaitre
Parameter**

(t) means “depends
on time”

Scale factor: compares the
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the past, and in the future

How are “Hubble’s Law” and the “expansion rate” related?



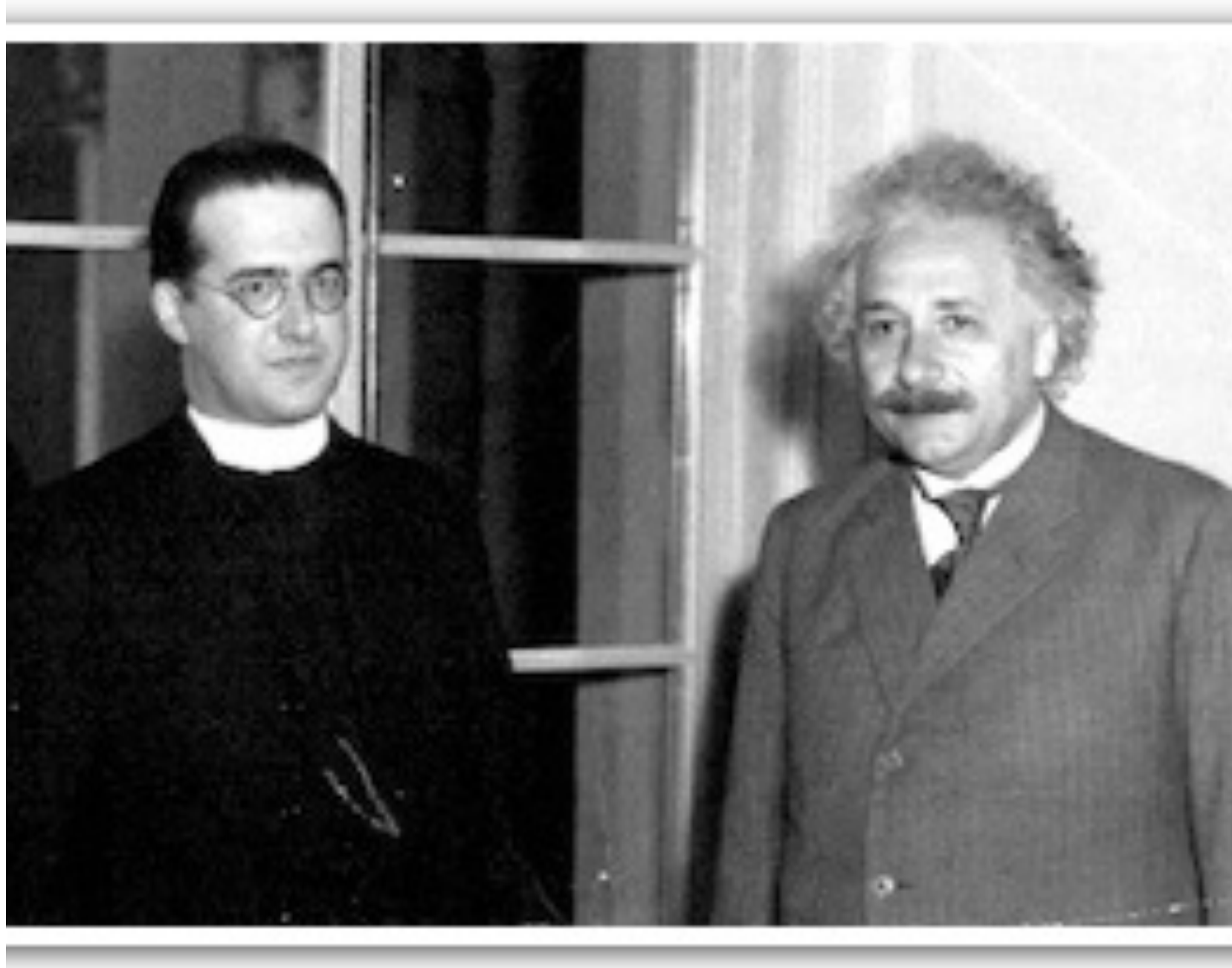
George Lemaitre to the rescue!

George Lemaitre and Einstein

How are “Hubble’s Law” and the “expansion rate” related?

George Lemaitre to the rescue!

In 1927, George Lemaitre, a Belgian priest and astrophysicist, **theoretically** derived Hubble’s Law and its connection to how quickly the scale factor of the universe is increasing! He did this **two years before** Edwin Hubble discovered his Famous Law through observation and analysis!



George Lemaitre and Einstein

$$V = H \times D$$

George Lemaitre 1927



George Lemaitre late 1920's

Interpreted Slipher's 21 out of 25 spiral nebula
receding away from us as the **physical expansion**
of space itself

George Lemaitre 1927

Suggested the universe was **not static** as Einstein had thought



George Lemaitre late 1920's

George Lemaitre 1927

Used Einstein's Field Equations to
derive equations for the expansion
of the universe theoretically



George Lemaitre late 1920's

George Lemaitre 1927

Assumptions



George Lemaitre late 1920's

George Lemaitre 1927

Assumptions

For large scales, say 300 Mly...



George Lemaitre late 1920's

George Lemaitre 1927

Assumptions

For large scales, say 300 Mly...

1. Universe was **filled with matter**



George Lemaitre late 1920's

George Lemaitre 1927

Assumptions

For large scales, say 300 Mly...

1. Universe was **filled with matter**

2. Looks the same no matter where you are... **Homogeneous** or spatially symmetric



George Lemaitre late 1920's

Assumptions

For large scales, say 300 Mly...

1. Universe was **filled with matter**
2. Looks the same no matter where you are...
Homogeneous or spatially symmetric
3. Looks the same no matter which direction you look... **Isotropic** or rotationally symmetric



George Lemaitre late
1920's

Assumptions

For large scales, say 300 Mly...

1. Universe was **filled with matter**
2. Looks the same no matter where you are...
Homogeneous or spatially symmetric
3. Looks the same no matter which direction you look... **Isotropic** or rotationally symmetric
4. The **peculiar motion** of galaxies through space was **negligible** compared to the expansion of space carrying the galaxies with them.



George Lemaitre late
1920's

Assumptions: Cosmological Principle

For large scales, say 300 Mly...

1. Universe was **filled with matter**
2. Looks the same no matter where you are...
Homogeneous or spatially symmetric
3. Looks the same no matter which direction you look... **Isotropic** or rotationally symmetric
4. The **peculiar motion** of galaxies through space was **negligible** compared to the expansion of space carrying the galaxies with them.



George Lemaitre late
1920's

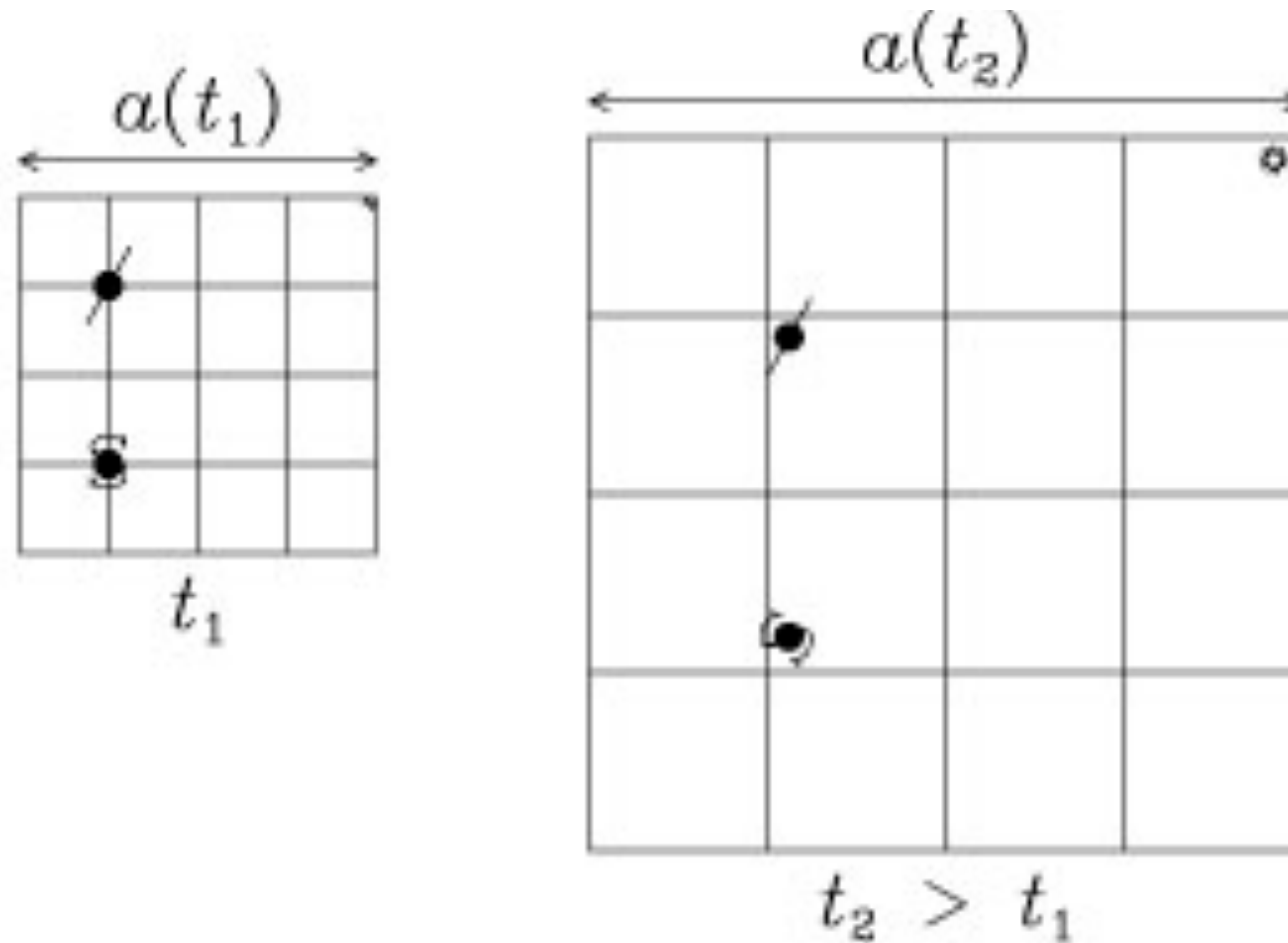
Theoretical framework?

For large scales, say 300 Mly...

He was first to set up the idea of scale factor $a(t)$ which depended on time and introduced comoving coordinates

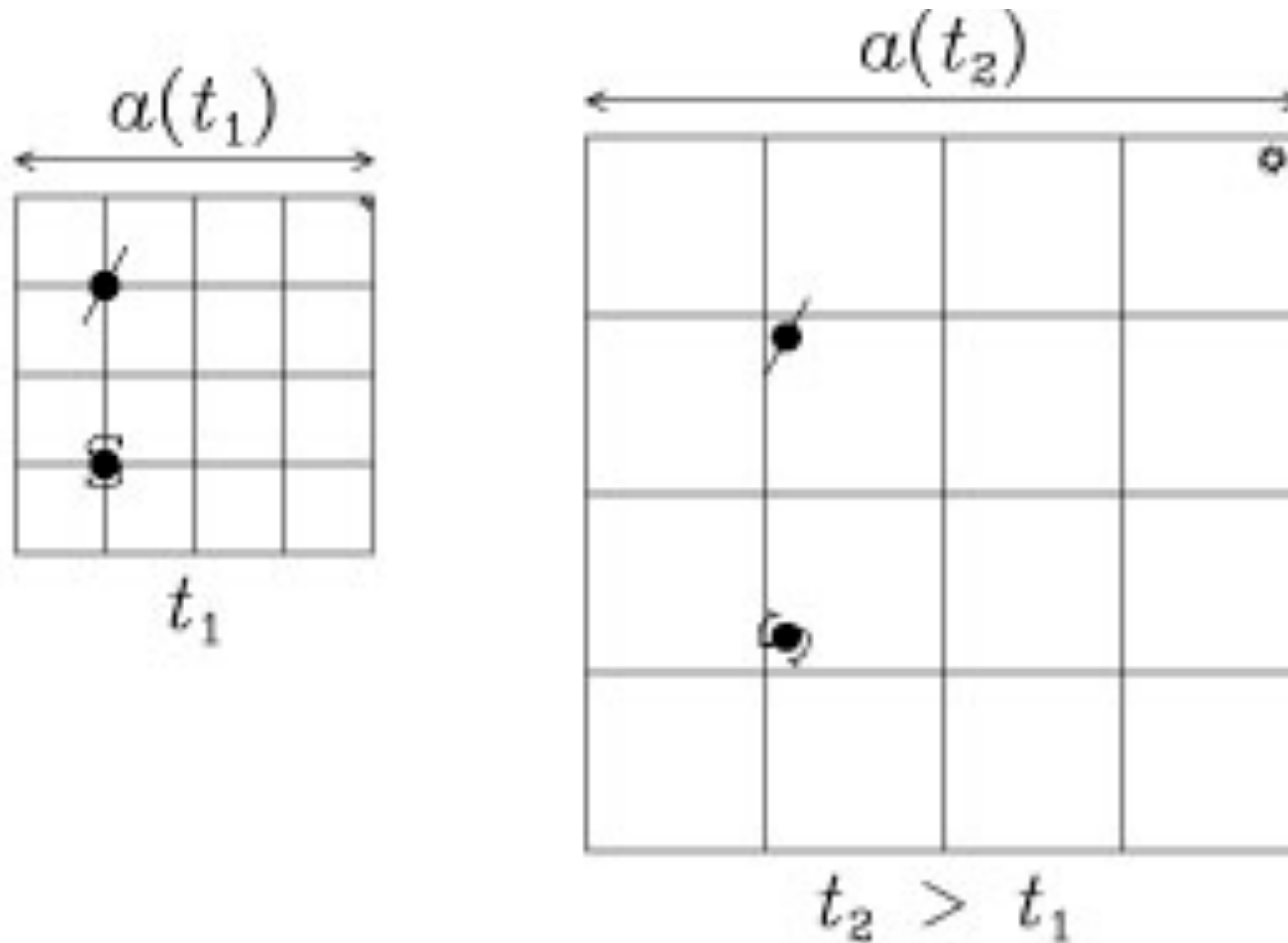


George Lemaitre late
1920's



Comoving coordinates and **Scale Factor**
 $a(t)$

The comoving coordinates expand with
the universe

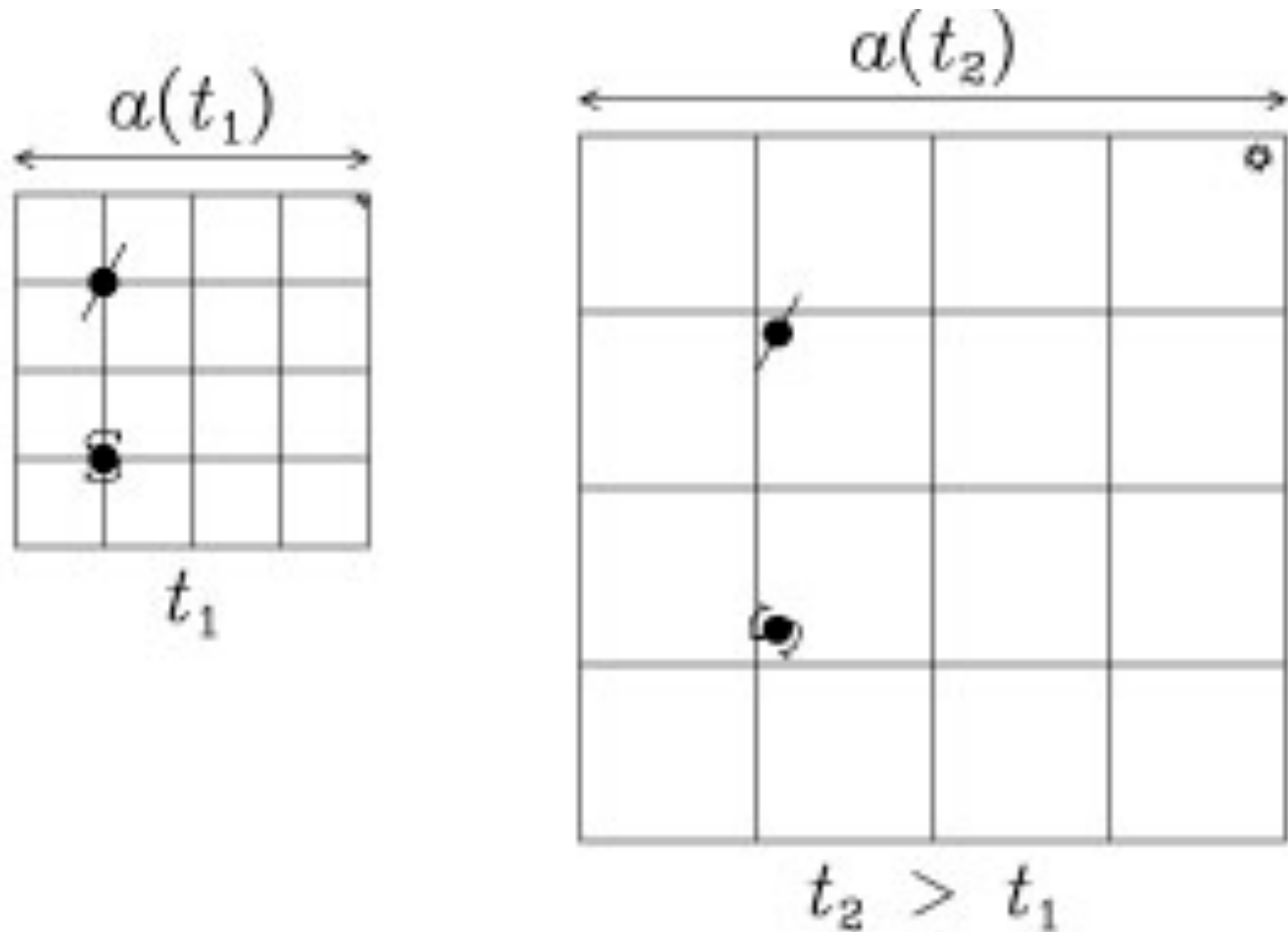


The comoving distance X between galaxies stays constant.

Here, the comoving distance between galaxies stays constant at $X = 2$ Mly (arbitrary unit).

Comoving coordinates and Scale Factor $a(t)$

The comoving coordinates expand with the universe



The comoving distance X between galaxies stays constant.

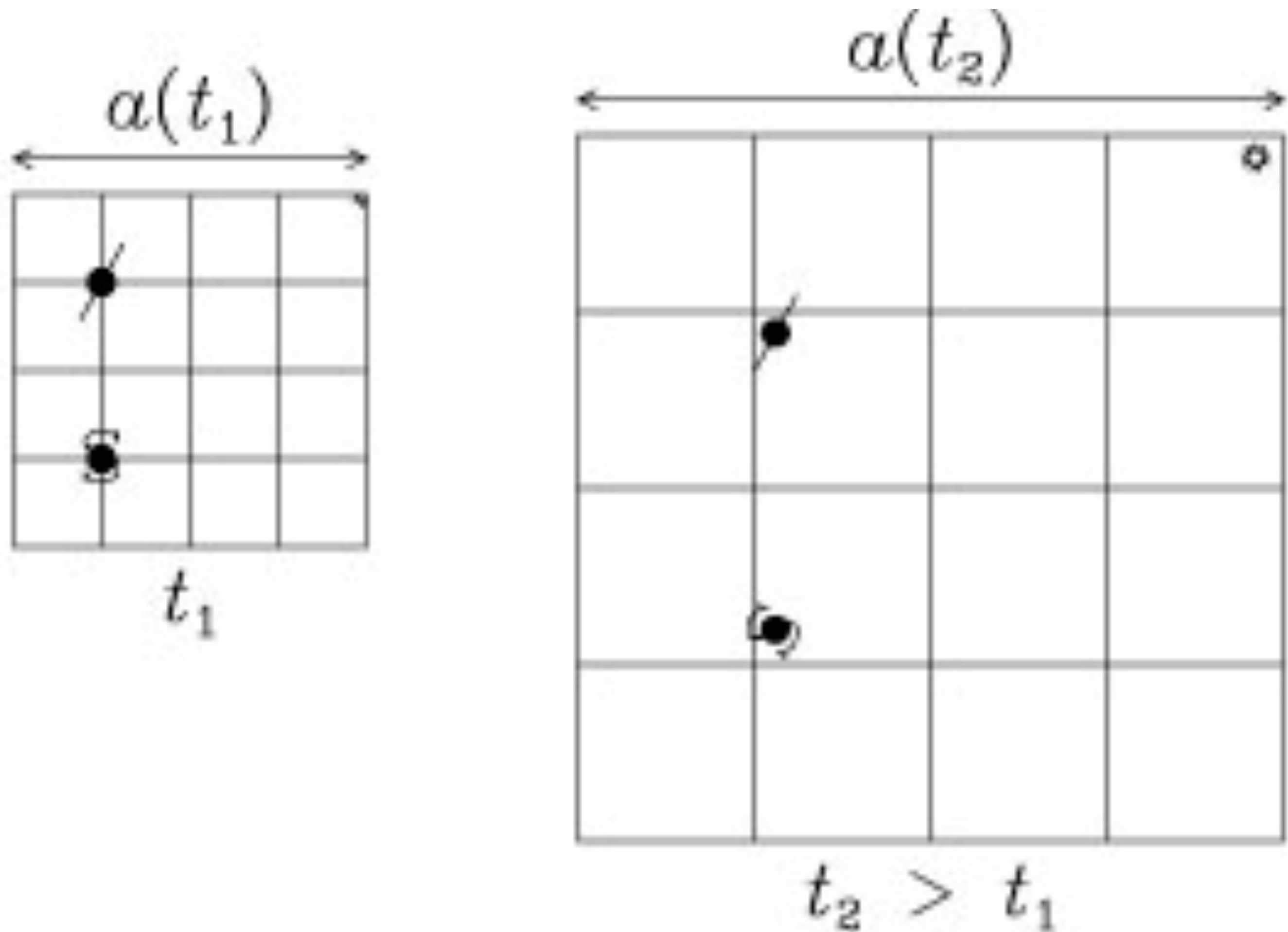
Here, the comoving distance between galaxies stays constant at $X = 2 \text{ Mly}$ (arbitrary unit).

The proper distance or actual tape measure distance at any one time does change

At any given time $D_p = a X$

Comoving coordinates and Scale Factor $a(t)$

The comoving coordinates expand with the universe



If at one cosmic time t_2 ,
 $a(t_2) = 5$

And $X = 2 \text{ MLY}$

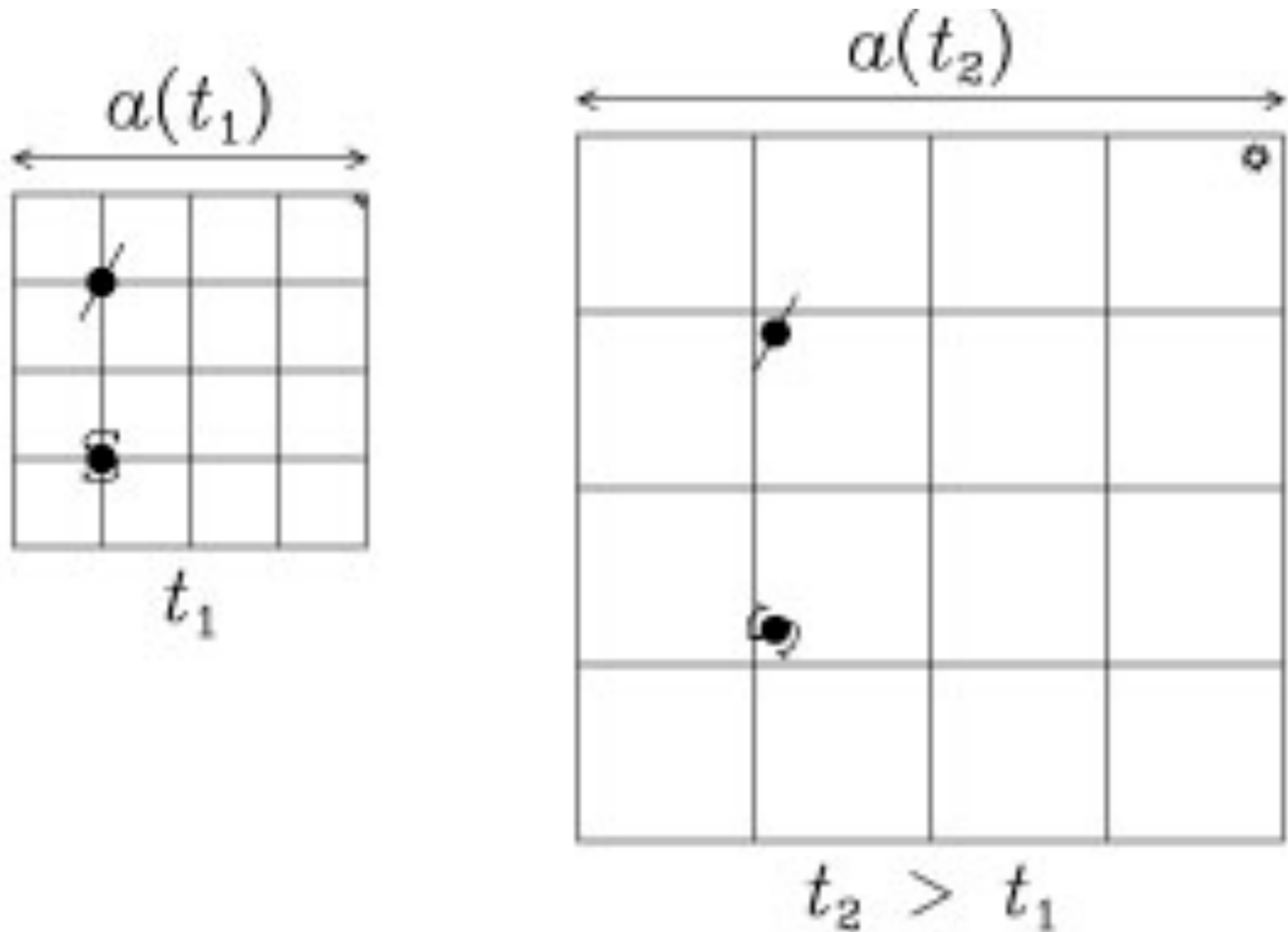
The **proper distance** at t_2

$$D_P = a X$$

$$D_P = 5 (2 \text{ MLY}) \\ = 10 \text{ MLY}$$

Comoving coordinates and **Scale Factor**
 $a(t)$

The comoving coordinates expand with
the universe



If at one cosmic time t_1 ,
 $a(t_1) = 3$

And $X = 2 \text{ MLY}$

The **proper distance** at t_1

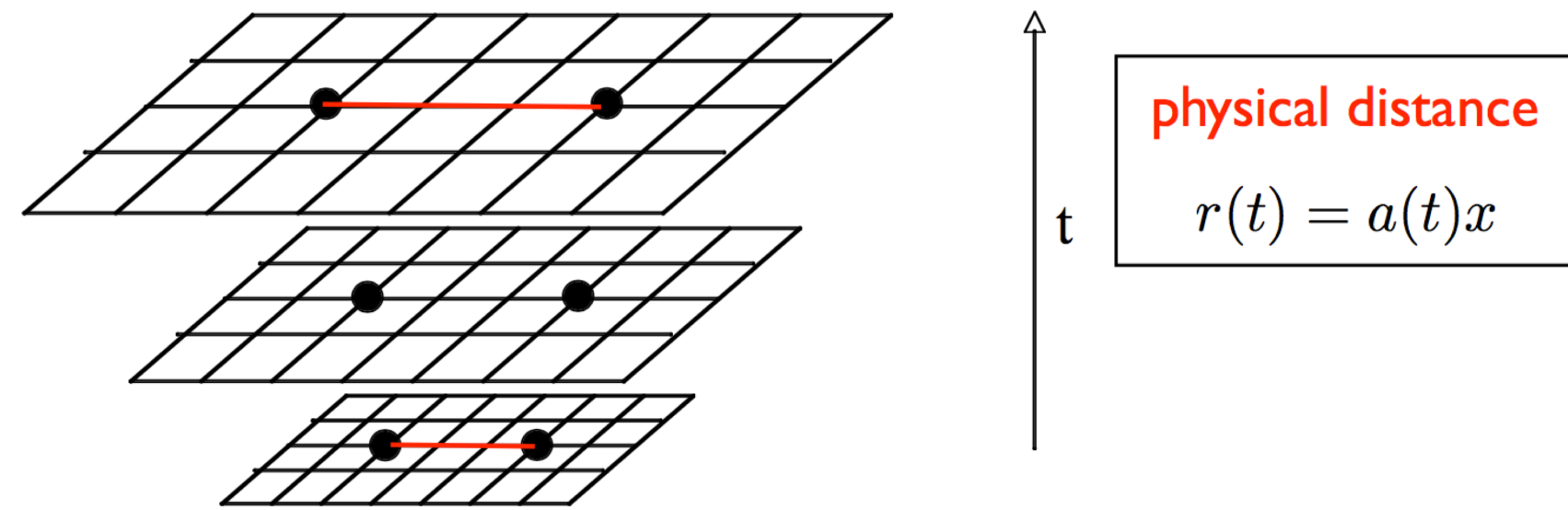
$$D_P = a X$$

$$D_P = 3 (2 \text{ MLY}) \\ = 6 \text{ MLY}$$

Comoving coordinates and **Scale Factor**
 $a(t)$

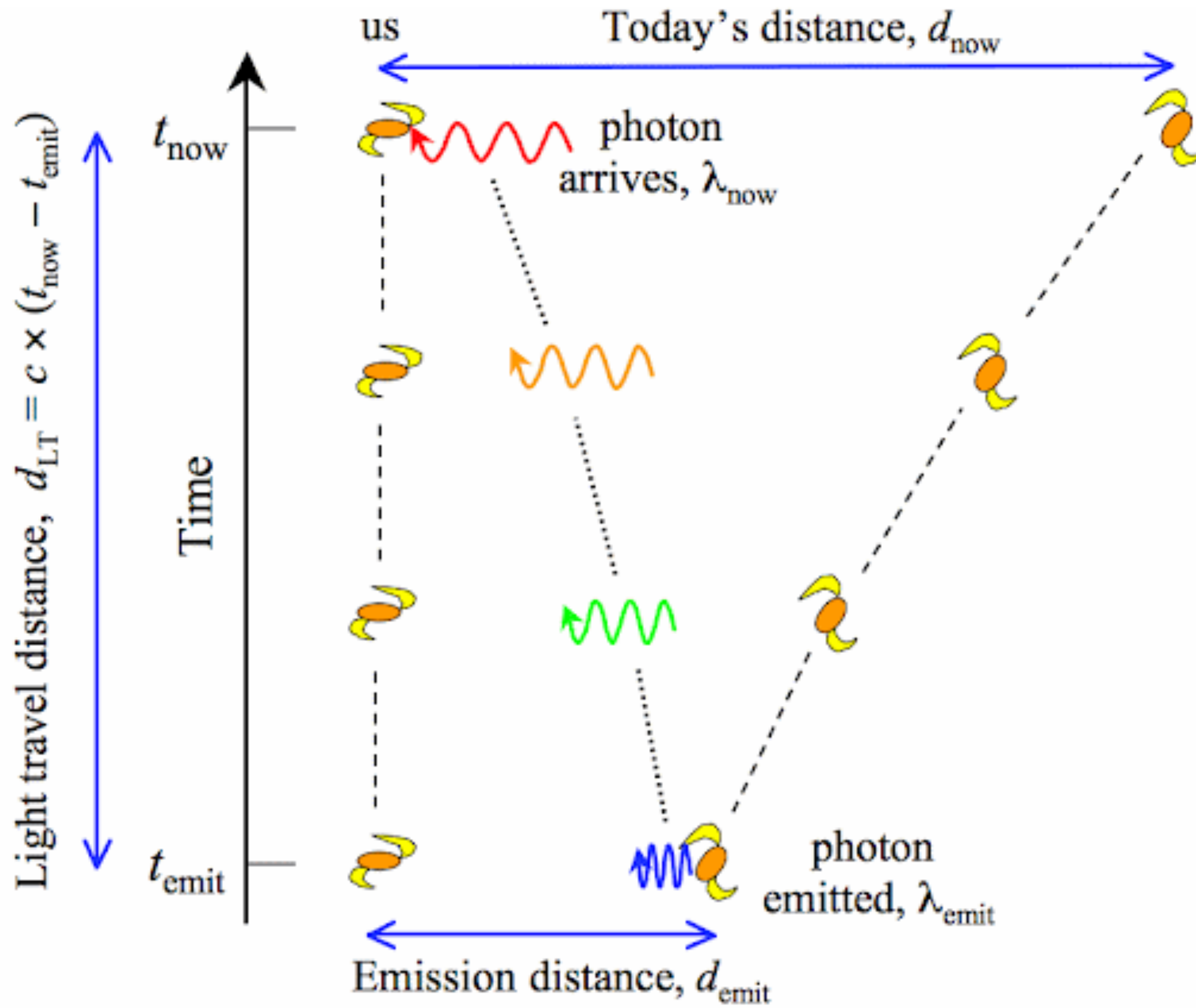
The comoving coordinates expand with
the universe

Distances: co-moving, physical, scale factor



co-moving distance x : distance measured on the grid (always 3 units in this example)

scale factor $a(t)$: size of the grid (varies with time as universe expands)



Another way of looking at proper distance or D_{Now}

Lemaitre's Derivation

Lemaitre's Derivation

$$r = a(t) X$$

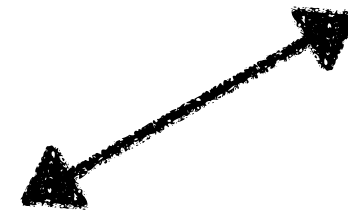
Lemaitre's Derivation

Proper distance

Comoving distance



$$r = a(t) X$$



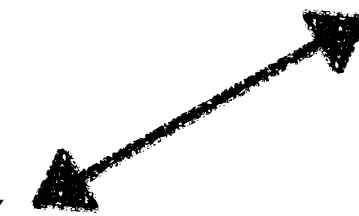
Lemaitre's Derivation

Proper distance

Comoving distance



$$r = a(t) X$$



Divide both sides by very small t value

Lemaitre's Derivation

Proper distance

Comoving distance


$$r = a(t) X$$

Divide both sides by very small t value

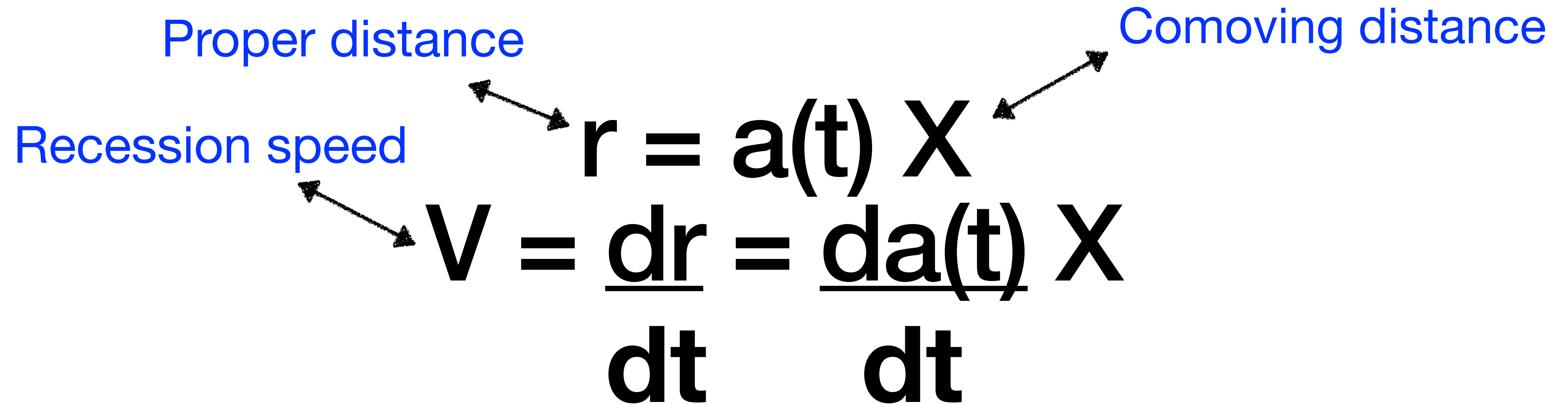
$$\frac{dr}{dt} = \frac{da(t)}{dt} X$$

Lemaitre's Derivation

Proper distance

Recession speed

Comoving distance

$$r = a(t) \times$$
$$V = \frac{dr}{dt} = \frac{da(t)}{dt} \times$$


Lemaitre's Derivation

Proper distance

Comoving distance

$$r = a(t) \times$$

Recession speed

Multiply top and
bottom by the
scale factor $a(t)$

$$V = \frac{da(t)}{dt} \times \frac{a(t)}{a(t)}$$

Lemaitre's Derivation

Proper distance

Comoving distance

$$r = a(t) \times$$

Recession speed

$$V = \frac{da(t)}{dt} \times a(t)$$

But what is
 $\times a(t)$?

Lemaitre's Derivation

Proper distance

Comoving distance

$$r = a(t) \times$$

It is just
proper
distance
 r

Recession speed

$$V = \frac{da(t)}{dt} \frac{r}{a(t)}$$

Lemaitre's Derivation

Proper distance

Comoving distance

$$r = a(t) X$$

Recession speed

$$V = \frac{da(t)}{dt} \frac{r}{a(t)}$$

Lets rewrite
the red in a
neater way!

Lemaitre's Derivation

Proper distance

Comoving distance

$$r = a(t) \times$$

Recession speed

Lets rewrite
the red in a
neater way!

$$V = \frac{da(t)/dt}{a(t)} r$$

Lemaitre's Derivation

Proper distance

Comoving distance

$$r = a(t) \times$$

Recession speed

Lets rewrite
the red in a
neater way!

$$V = \frac{da(t)/dt}{a(t)} r$$

For an expanding universe
following Einstein's Field
equations and his
cosmological principle,
Lemaitre also knew
theoretically that $V \propto D$
i.e. Hubble's law

$$V = H(t) r$$

Lemaitre's Derivation

Proper distance

Comoving distance

$$r = a(t) X$$

Recession speed

$$V = \frac{da(t)/dt}{a(t)} r$$

Compare the
two bottom
equations.
What is $H(t)$?

For an expanding universe
following Einstein's Field
equations and his
cosmological principle,
Lemaitre also knew
theoretically that $V \propto D$
i.e. Hubble's law

$$V = H(t) r$$

$$H(t) = \frac{da(t)}{a(t) dt}$$

**This defines what the
Hubble -Lemaitre parameter
is all about!**

Redshift and Scale Factor

The scale factor and redshift are related in a simple equation. Let's reason it out!

Redshift and Scale Factor

$$Z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

Redshift and Scale Factor

$$Z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

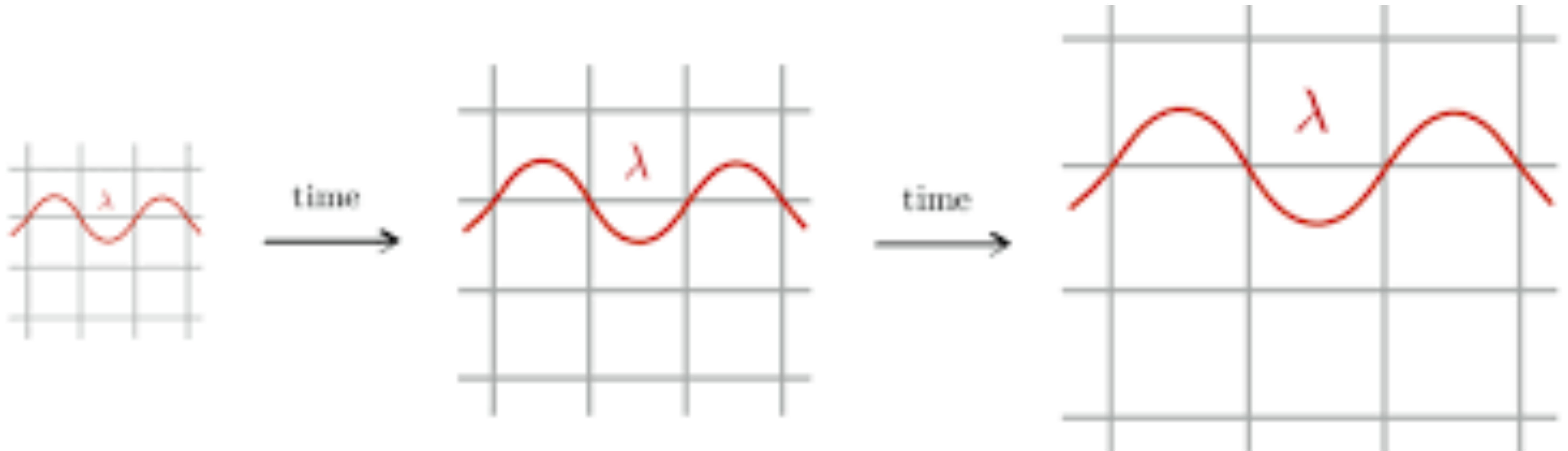
$$Z = \lambda_{\text{obs}}/\lambda_{\text{em}} - 1$$

Redshift and Scale Factor

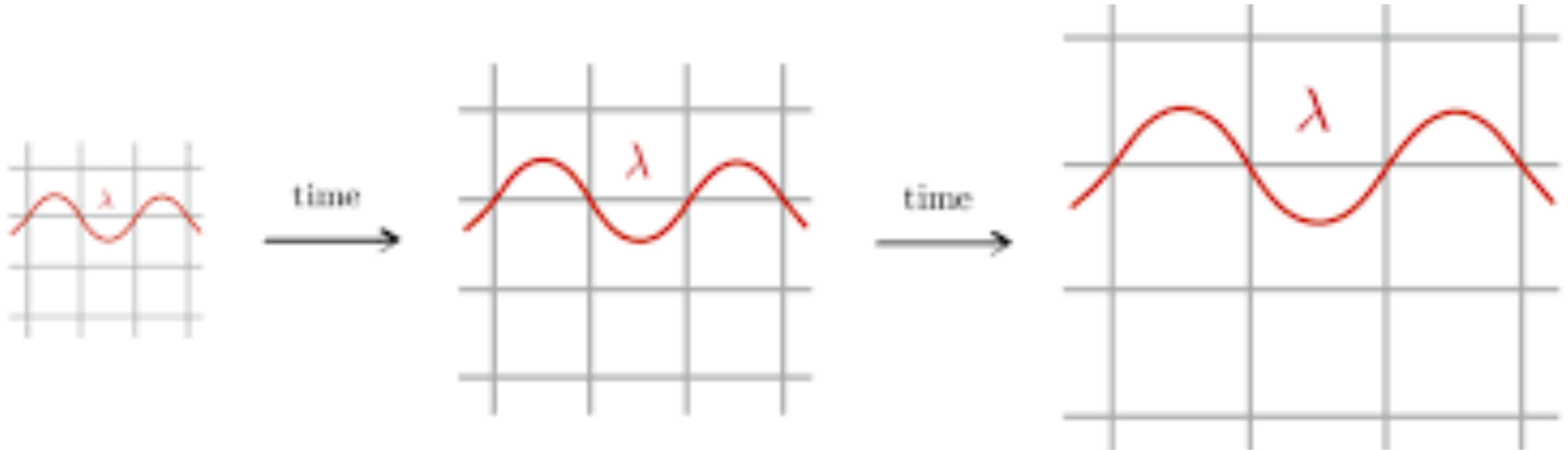
$$Z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

$$Z = \lambda_{\text{obs}}/\lambda_{\text{em}} - 1$$

$$Z + 1 = \lambda_{\text{obs}}/\lambda_{\text{em}}$$

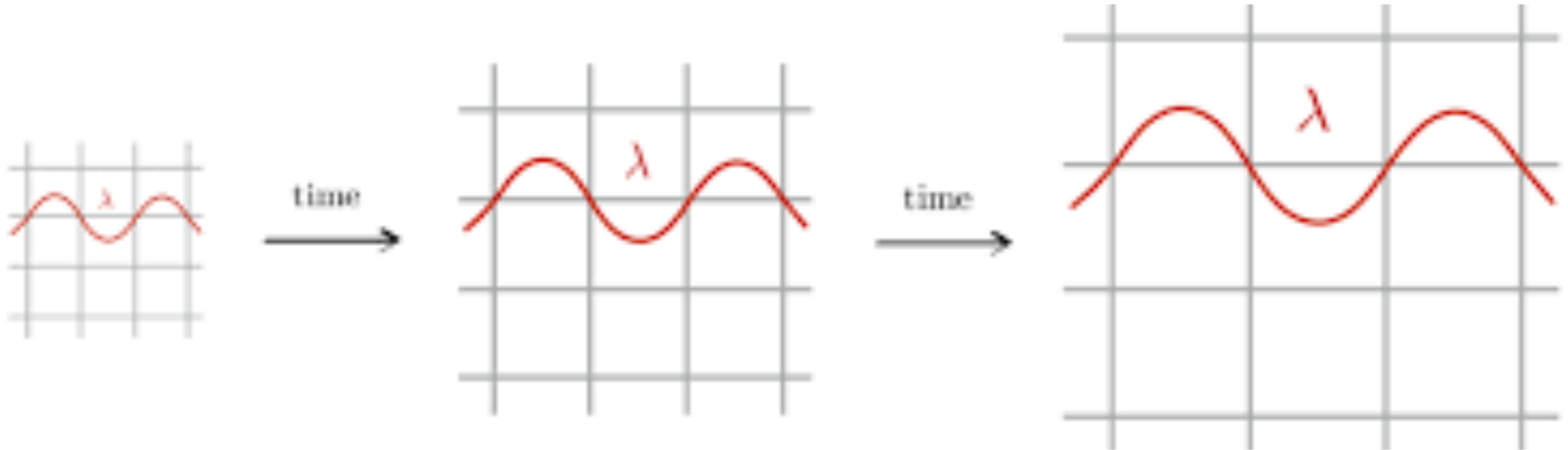


Light gets stretched by expanding space!



Light gets stretched by expanding space!

So a “ is proportional to ” λ



Light gets stretched by expanding space!

So a “ is proportional to ” λ

Which means... $a(t_{\text{obs}}) / a(t_{\text{em}}) = \lambda_{\text{obs}} / \lambda_{\text{em}}$

Redshift and Scale Factor

$$Z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

$$Z = \lambda_{\text{obs}}/\lambda_{\text{em}} - 1$$

$$Z + 1 = \lambda_{\text{obs}}/\lambda_{\text{em}} \quad \text{red symbols can replace with}$$
$$a(t_{\text{obs}}) / a(t_{\text{em}})$$

Redshift and Scale Factor

$$Z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

$$Z = \lambda_{\text{obs}} / \lambda_{\text{em}} - 1$$

$$Z + 1 = a(t_{\text{obs}}) / a(t_{\text{em}})$$

Redshift and Scale Factor

Common Formula in Astrophysics

$$Z + 1 = a(t_{\text{obs}}) / a(t_{\text{em}})$$

Redshift and Scale Factor

Common Formulas in Astrophysics

$$Z + 1 = a(t_{\text{obs}}) / a(t_{\text{em}})$$

By convention, $a(t_{\text{obs}}) = 1$

Therefore $Z + 1 = 1 / a(t_{\text{em}})$

Redshift and Scale Factor

Common Formulas in Astrophysics

$$Z + 1 = a(t_{\text{obs}}) / a(t_{\text{em}})$$

By convention, $a(t_{\text{obs}}) = 1$

Therefore $Z + 1 = 1 / a(t_{\text{em}})$

Knowing redshift can get us the scale factor in the past!